

NYU, Apr 2025

# QUASIPARTICLES, FLAT BANDS, AND EXOTIC TOPOLOGY IN HYDRODYNAMIC MATTER

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TT. [Exceptional topology in ordinary soft matter](#). *PRE* 2021.

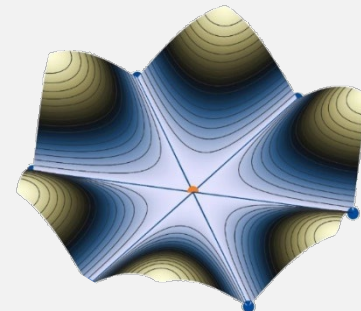
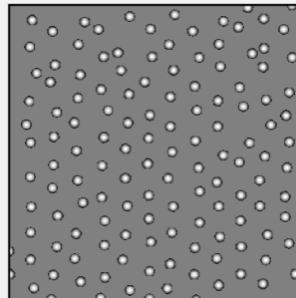
Saeed, Pak et al. [Quasiparticles, flat bands and the melting of hydrodynamic matter](#). *Nature Physics* 2023 <sup>1</sup>

Collective phenomena in  
classical dissipative systems:  
Non-eq. transitions

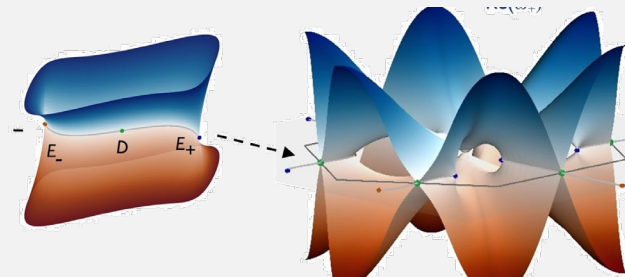


Collective phenomena in  
quantum matter:  
quasiparticles, flat bands...

1. Non-eq. transitions in a  
hydrodynamic crystal  
(Hermitian)



2. Exceptional topology by coupling  
elasticity and hydrodynamics.  
(non-Hermitian)

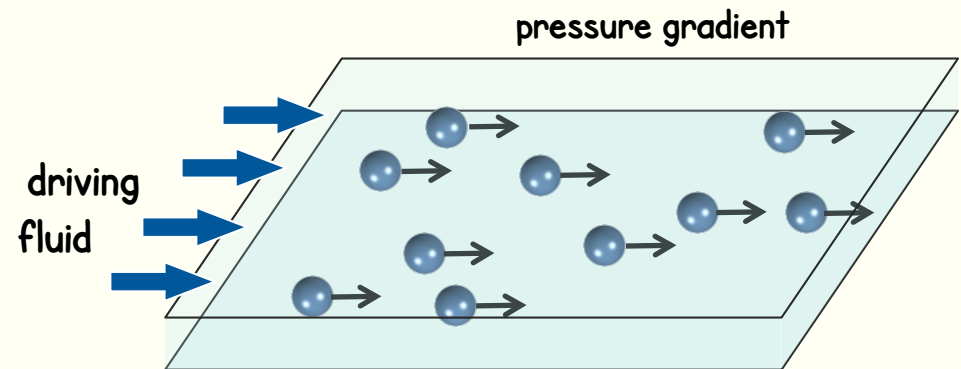


*Please ask questions during the talk*



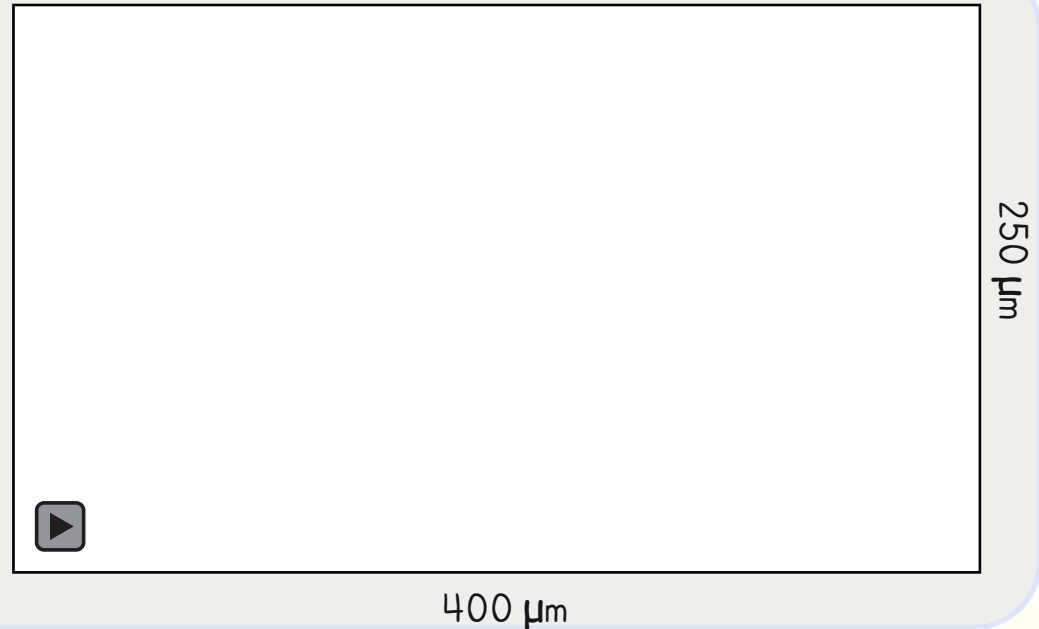
A CLASSICAL DISSIPATIVE MODEL SYSTEM:

2D ensemble of micro-particles  
driven in a thin sheet of fluid

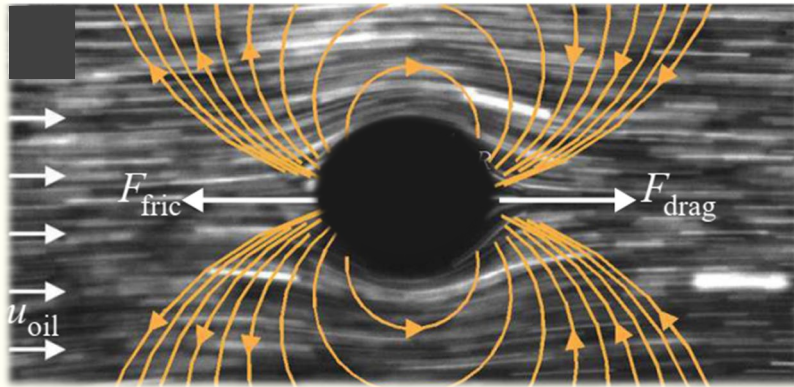


Microfluidic channel with driven colloids:

- System is **dilute**: 1.8% area density.
- **Effectively 2D**: Diameter 7  $\mu\text{m}$ , height - 10  $\mu\text{m}$
- **Friction**: velocity relative to fluid: 100  $\mu\text{m/s}$



## Particles driven in a quasi-2D geometry induce hydrodynamic dipoles



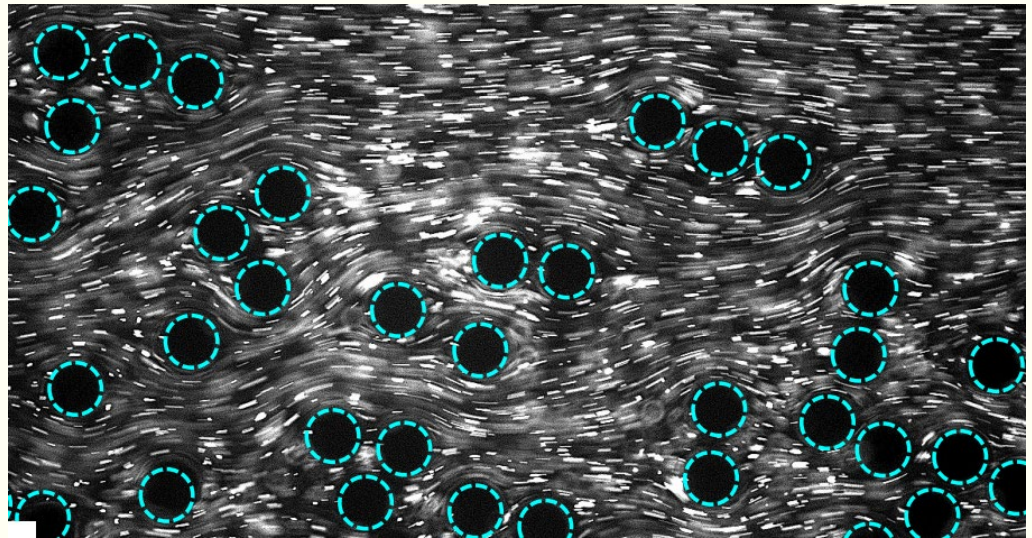
Beatus, Bar-Ziv et al. (Nature Phys 2006, 2014)

Reynolds number:  $10^{-4} \rightarrow$  forget inertia.

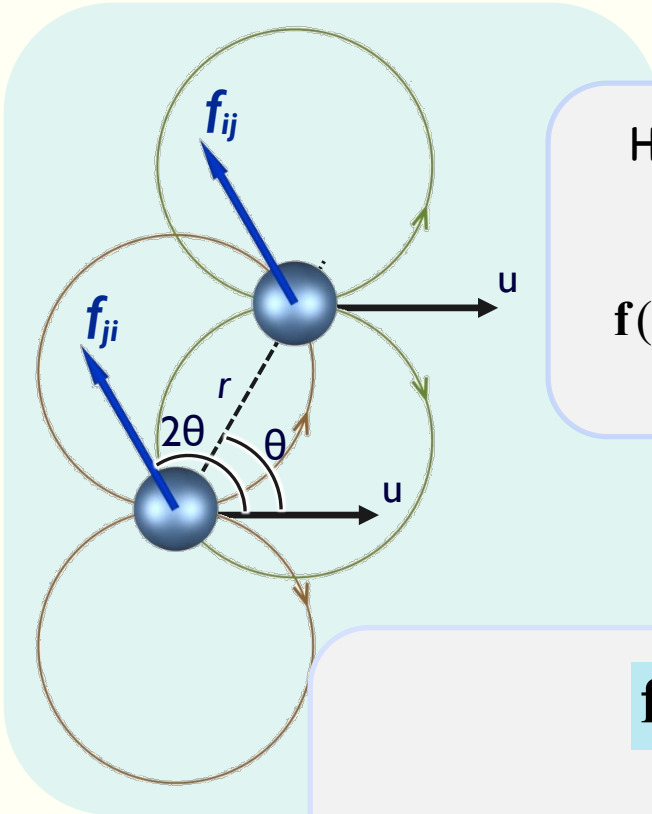
Peclet number  $10^4 \rightarrow$  also thermal forces.

$$\mathbf{F}_{\text{fric}} = \gamma \dot{\mathbf{r}}_i = \mathbf{F}_{\text{drag}}$$

Particles interact by  
perturbing streamlines and  
thereby inducing drag forces.



## Twofold symmetry of the forces leads to invariance under parity



Hydrodynamic dipole

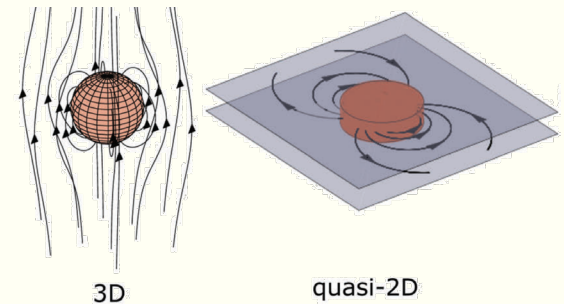
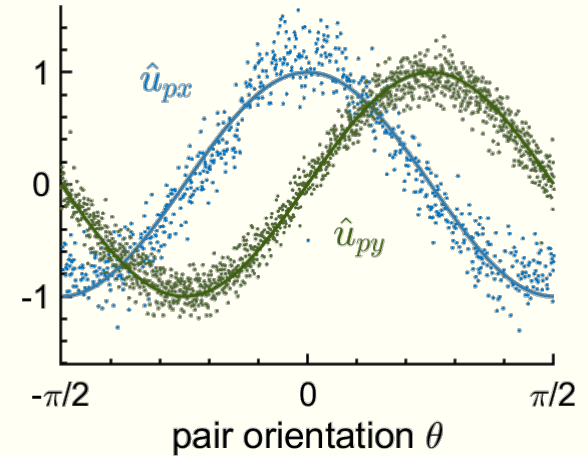
$$\mathbf{f}(\mathbf{r}) \sim \frac{1}{r^2} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$$

$$\mathbf{f}(-\mathbf{r}) = \mathbf{f}(\mathbf{r})$$

Two particles exert on each other **EQUAL FORCES**.

[ Unlike Newton's 3<sup>rd</sup> law:  $\mathbf{f}(-\mathbf{r}) = -\mathbf{f}(\mathbf{r})$  ]

Measured pair velocity



$$v \sim \frac{R}{r} + \left(\frac{R}{r}\right)^3$$

Momentum  
monopole

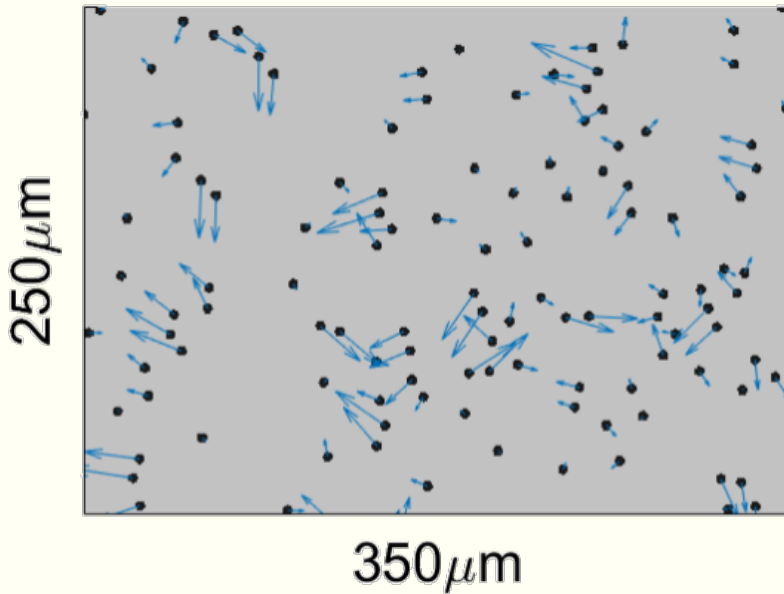
$$v \sim \left(\frac{R}{r}\right)^2$$

Mass dipole

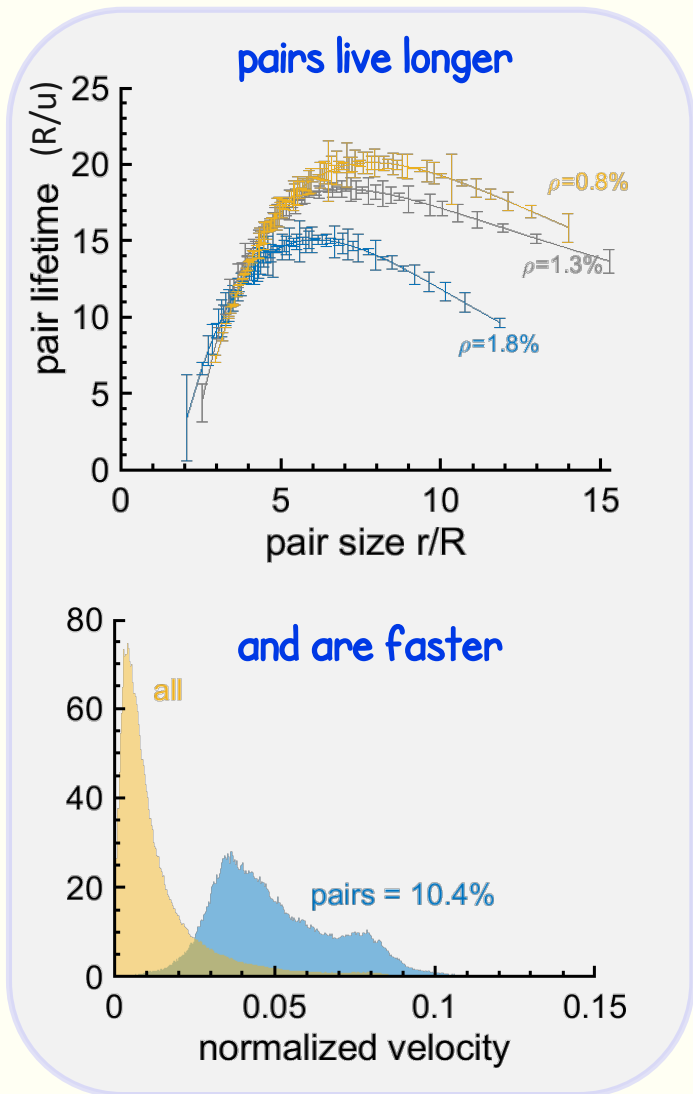


*This is the origin of all physical phenomena we will discuss*

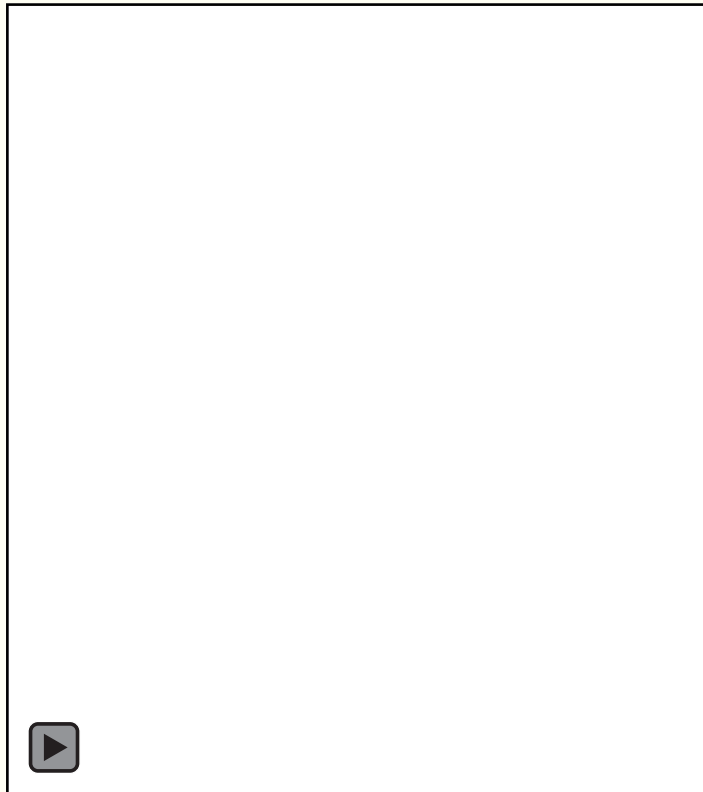
Weakly-interacting pairs are stable between collisions



*Are pairs elementary particle-like excitations?*



## Long-range forces excite phonons in the hydrodynamic crystal



Simulation 51 x 51 particles



*Please ask questions*

Equations of motion  $\gamma \dot{\mathbf{r}}_i = \sum_j \mathbf{f}_{ij} + \mathbf{F}$

Uniform steady-state at  $\dot{\mathbf{r}}_i = \mathbf{F} / \gamma$

Normal modes, "phonons", around steady state

$$\delta \mathbf{r}_i = \boldsymbol{\psi}_k e^{i(\mathbf{k} \cdot \bar{\mathbf{r}}_i - \omega t)}$$

2D polarization  $\boldsymbol{\psi}_k$  is an eigenvector of a "Schrödinger" equation

$$\omega_k \boldsymbol{\psi}_k = H \boldsymbol{\psi}_k$$

with a **TRIPOLAR** "spring constant"

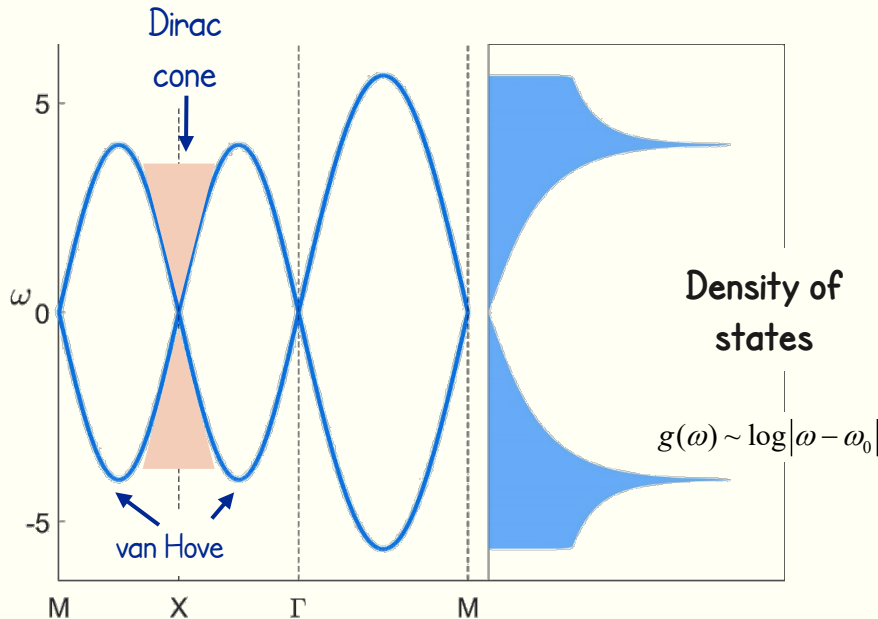
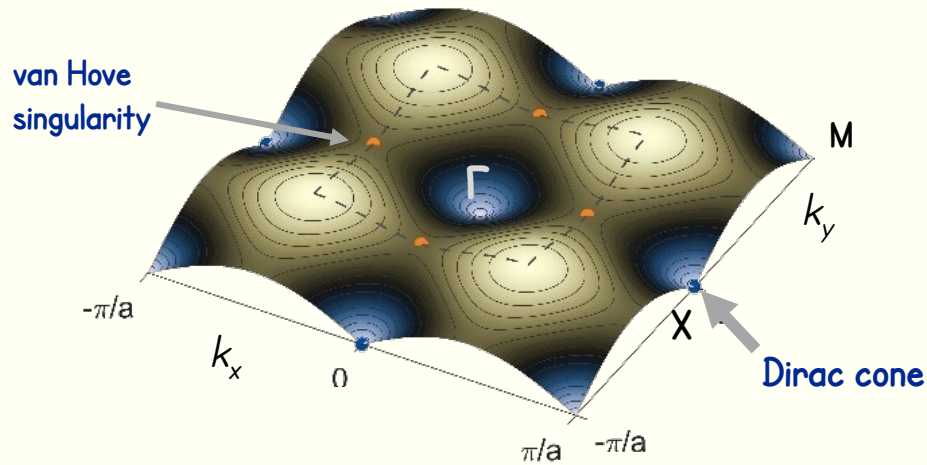
$$H = \boldsymbol{\Omega} \cdot \boldsymbol{\sigma},$$

$$\boldsymbol{\Omega} = \begin{pmatrix} \Omega_x \\ \Omega_y \end{pmatrix} \sim \sum_j \frac{e^{i\mathbf{k} \cdot \bar{\mathbf{r}}_j}}{\bar{\mathbf{r}}_j^3} \begin{pmatrix} \cos 3\theta_j \\ \sin 3\theta_j \end{pmatrix}.$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix}. \quad \text{Pauli's matrices}$$

# The spectrum of hydrodynamic phonons hints for quasiparticles

1<sup>st</sup> Brillouin zone



Spectrum is purely real with propagating phonons

$$\omega_{\mathbf{k}} = \pm |\mathbf{\Omega}| = \pm \sqrt{\Omega_x^2 + \Omega_y^2}$$

In a square lattice with nearest neighbors

$$\omega_{\mathbf{k}} \sim \pm \sqrt{2 - \cos 2k_x a - \cos 2k_y a}$$

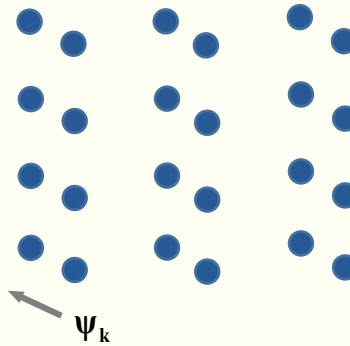
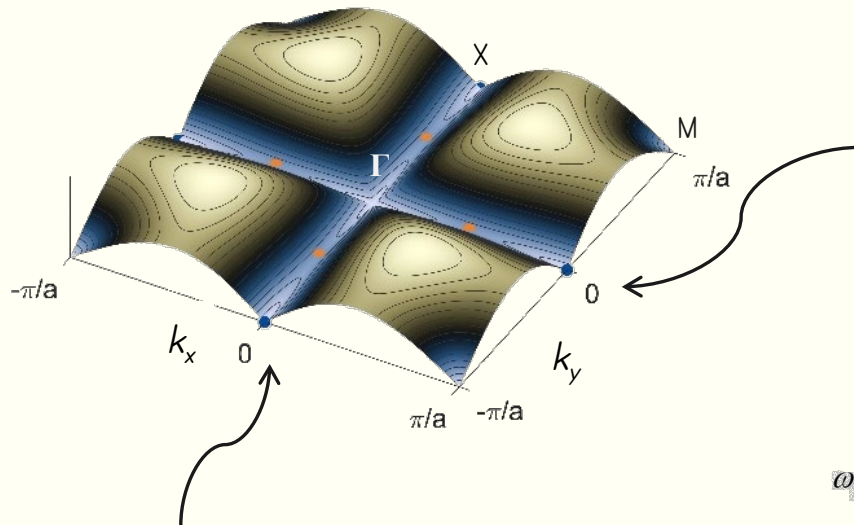
THREE HALLMARKS:

- **Dirac cones** (X).
- van Hove singularities *inside* Brillouin zone.
- **Smaller** primitive cell (period =  $2a$ ).



# Excitations at the Dirac points are pairing modes

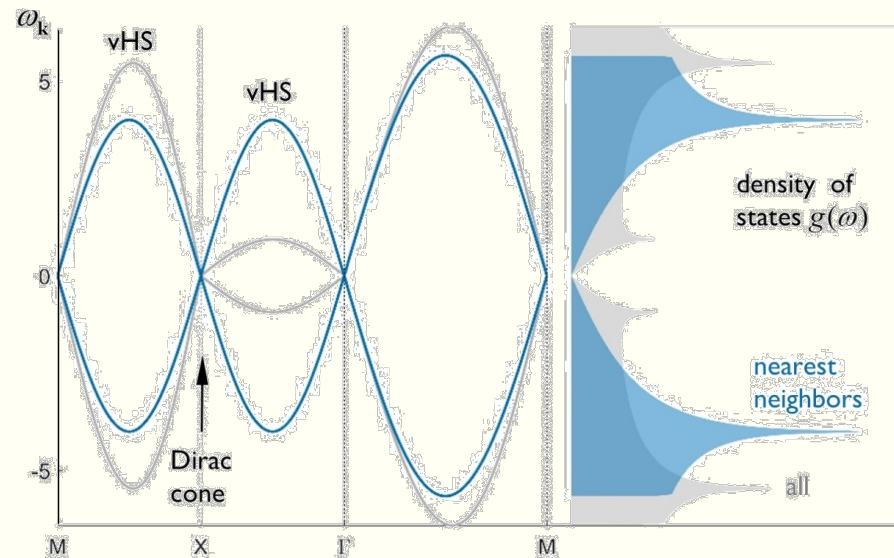
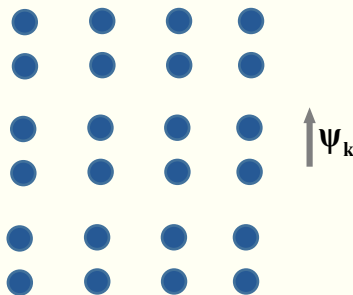
all neighbors



Dirac cone = acoustic modes of pairing lines

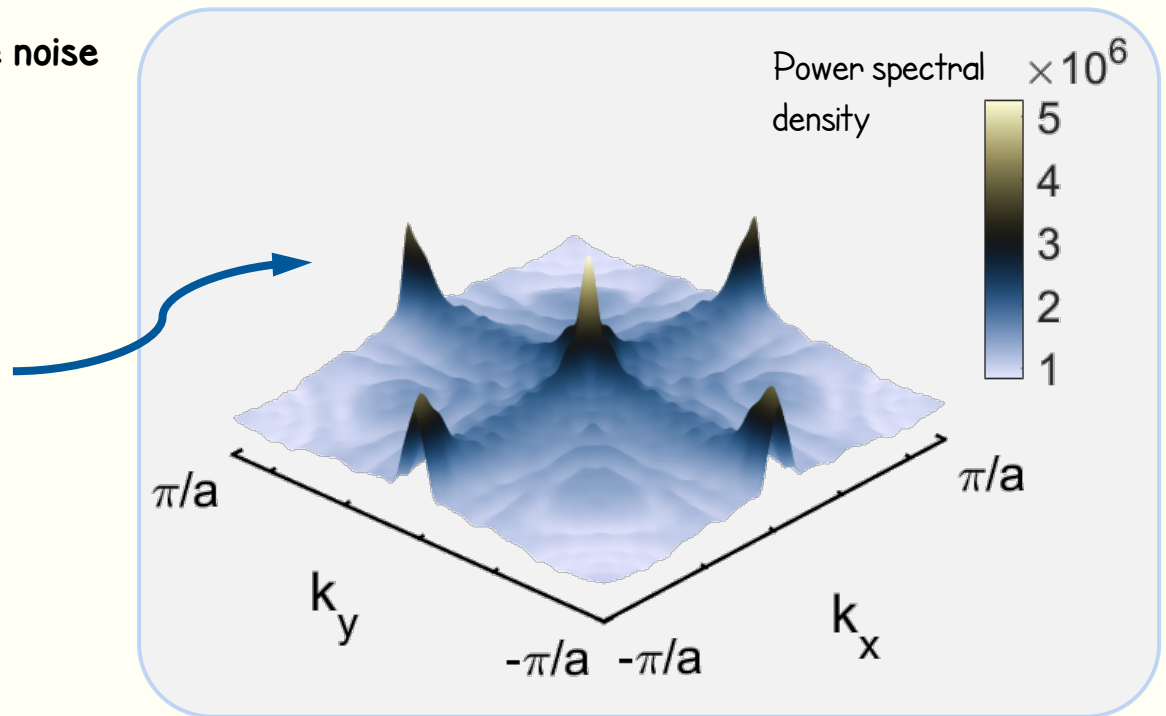
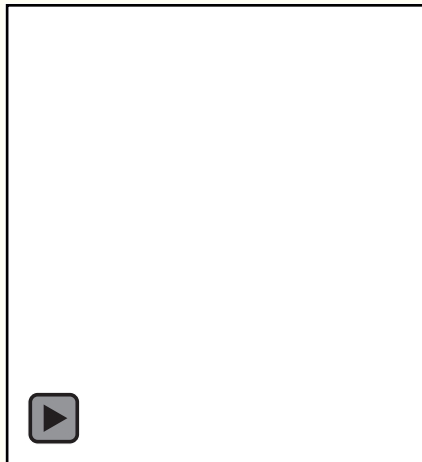
$$\omega \sim c \cdot (\mathbf{k} - \mathbf{k}_D)$$

pairing modes



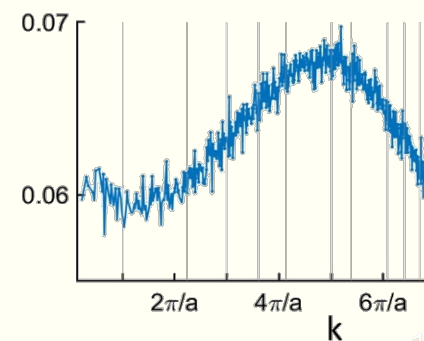
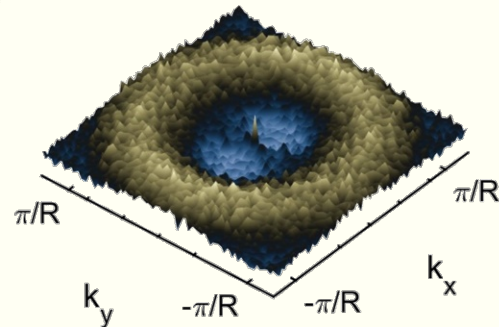
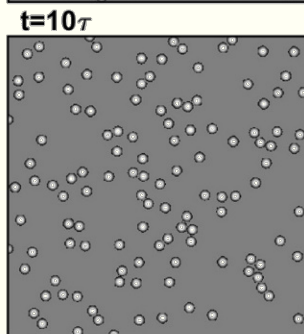
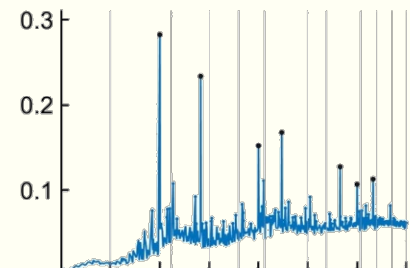
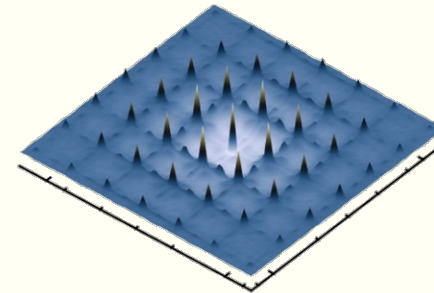
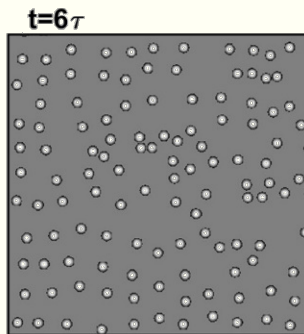
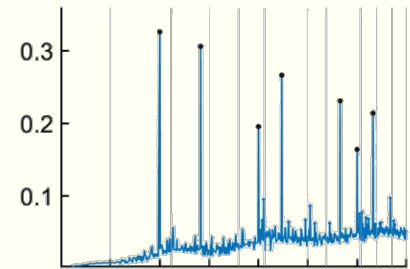
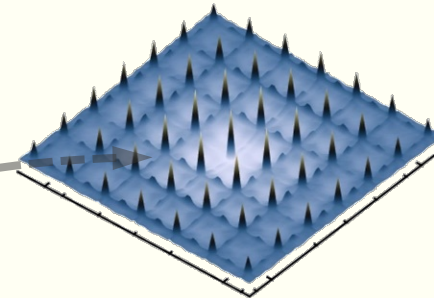
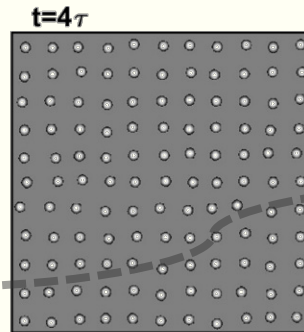
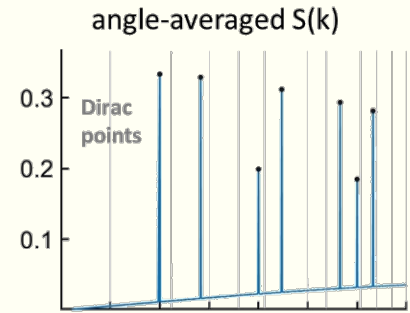
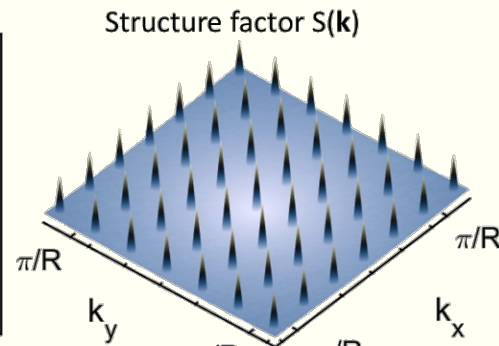
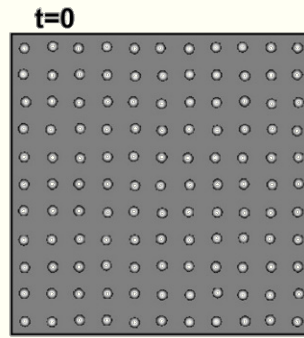
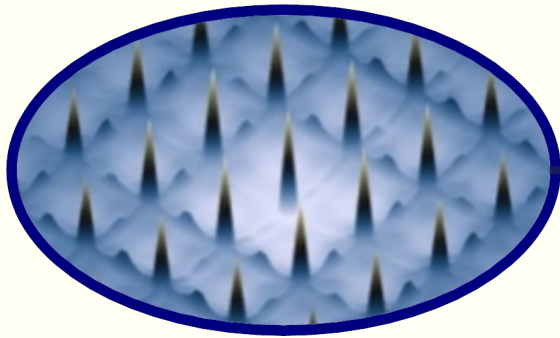
## Pairing modes emerge via *nonlinear* mode coupling (phonon-phonon processes)

Simulation starting from white noise





# Pairs induce the melting of the hydrodynamic crystal



Natural timescale in crystal

$$\tau = \frac{a}{\text{sound speed}} \sim \frac{a^3}{uR^2}$$



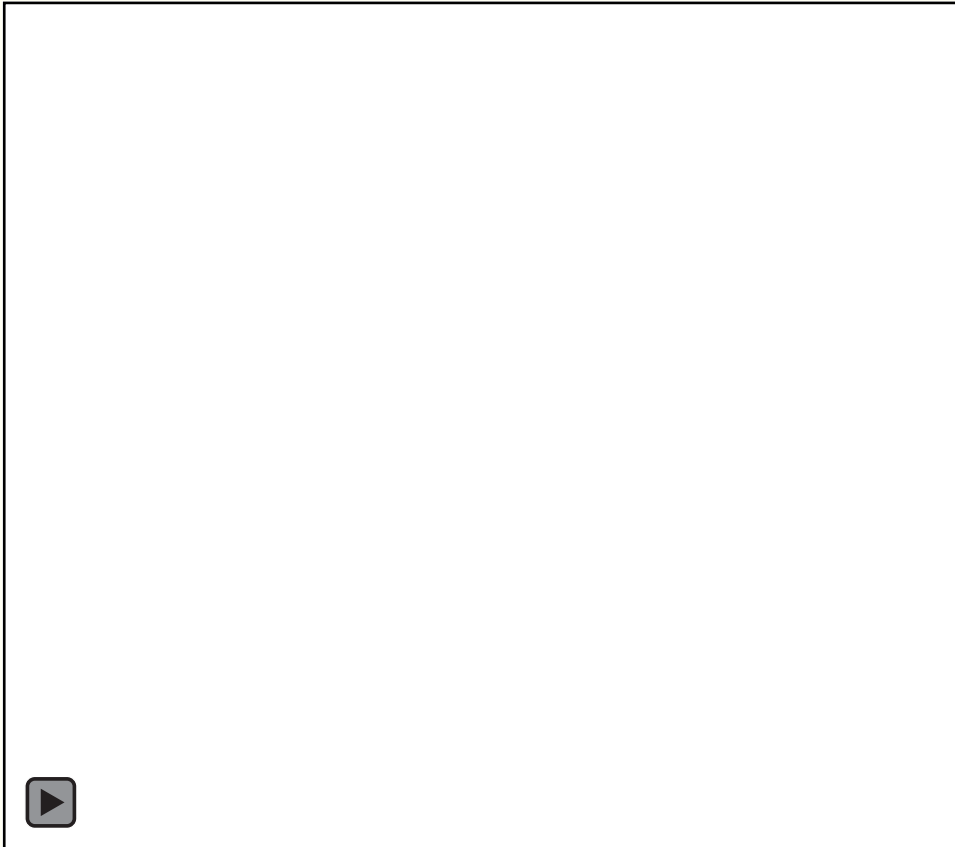
Questions?

Pairs induce the melting of the hydrodynamic crystal

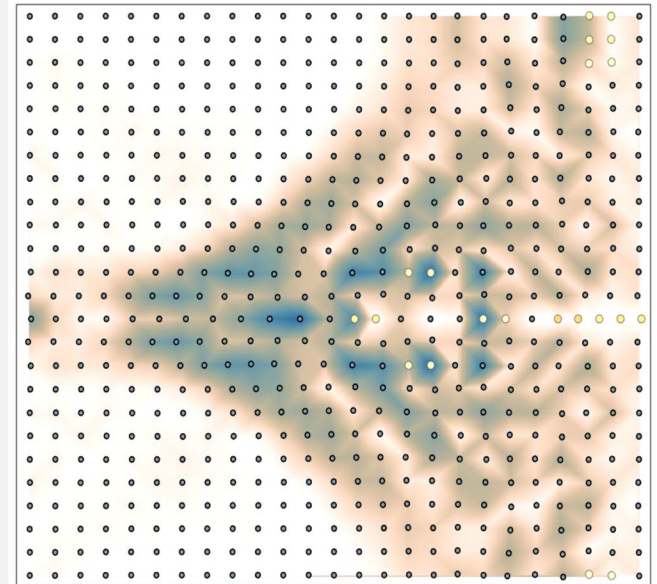


# Quasiparticles excite supersonic pairing avalanches, leading to melting

Localized perturbation ("Green function")



$t = 2.87\tau$



Mach cone of pairing trailing a quasiparticle.

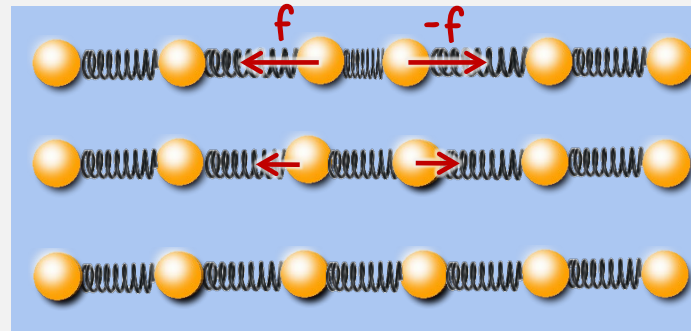
Mach # = 1.4 (shading indicates velocity)

Recap:  $f(-r) = f(r)$  parity makes quasiparticles and phonons propagate freely

In equilibrium crystals (e.g. elastic):

force  $\sim \nabla \text{strain} \sim \nabla^2 \text{displacement}$

$$\gamma \dot{u} = \kappa \nabla^2 u \Rightarrow \omega \sim i \sqrt{\frac{\kappa}{\gamma}} k^2$$



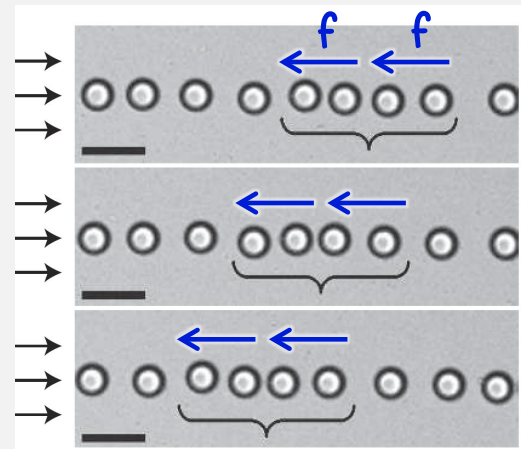
waves and disturbances diffuse and dissipate;  
crystal relaxes to equilibrium

In hydrodynamic crystals:

force  $\sim \text{strain} \sim \nabla \text{displacement}$

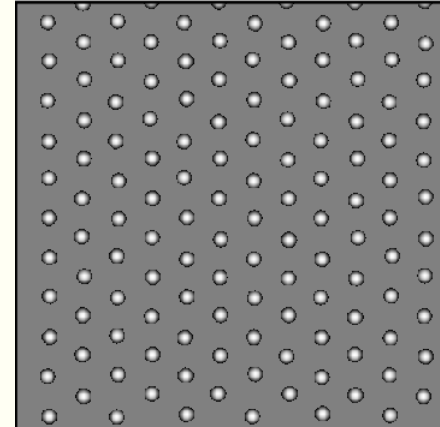
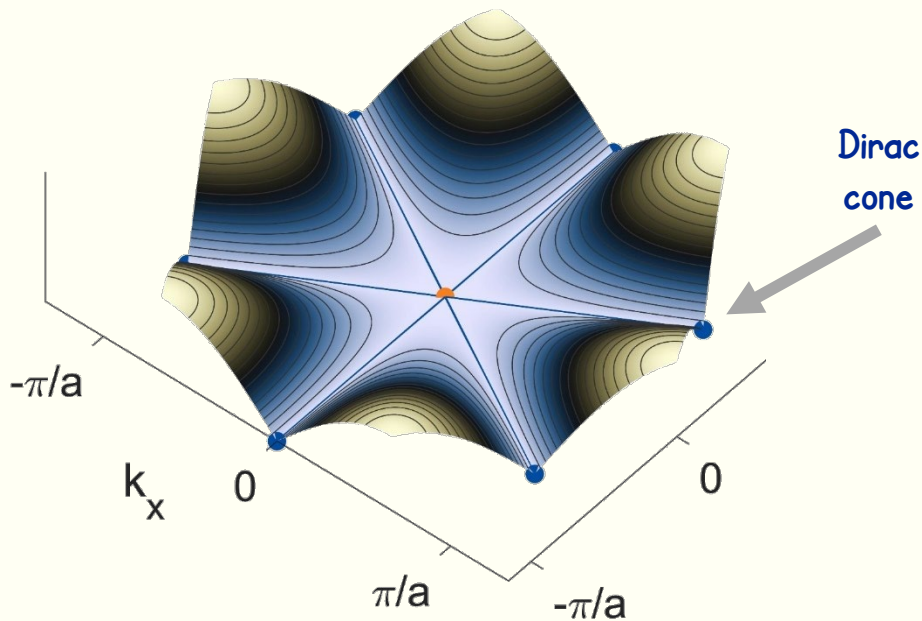
$$\gamma \dot{u} = h \nabla u \Rightarrow \omega \sim \left(\frac{h}{\gamma}\right) k = c k$$

waves and disturbances propagate without damping



## SPECIAL CASE

Threefold symmetry of the hydrodynamic interaction matches the symmetry of a hexagonal crystal



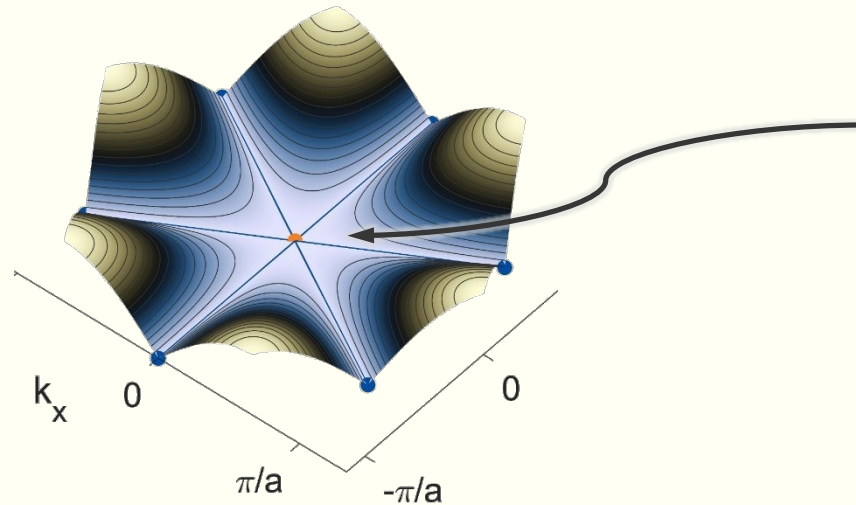
In a hexagonal lattice with nearest neighbors

$$\omega_k \sim \pm \left| \sin \frac{1}{2} k_x a \right| \left| \cos \frac{1}{2} k_x a - \cos \frac{\sqrt{3}}{2} k_y a \right|$$

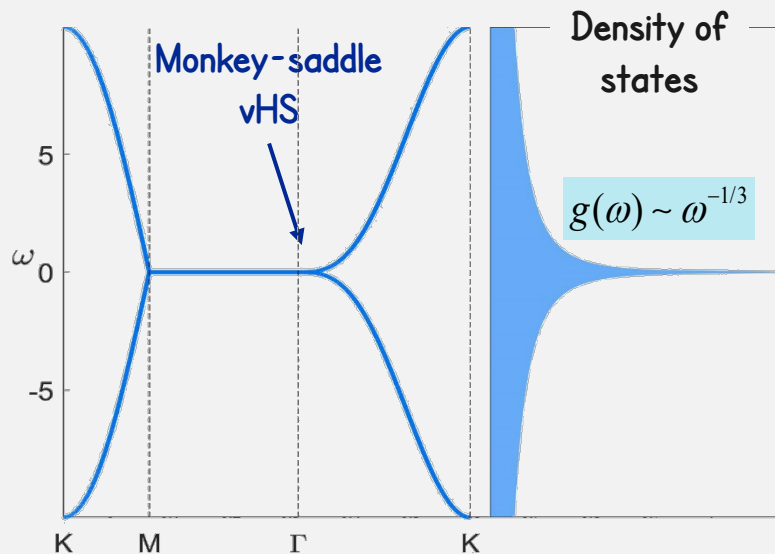
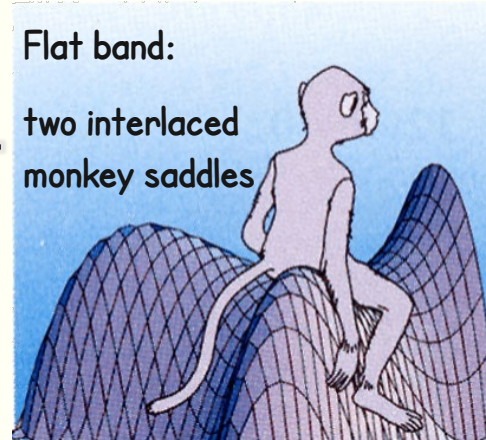
HALLMARKS:

- Single van Hove singularity at  $k=0$  ( $\Gamma$ )
- Dirac points (M) extend into 1D zero-frequency lines (cones  $\rightarrow$  "wedges").

# Hexagonal crystals exhibit a nearly-flat band of slow excitations



Flat band:  
two interlaced  
monkey saddles



The van Hove singularity is a multicritical "monkey saddle" with divergent d.o.s.

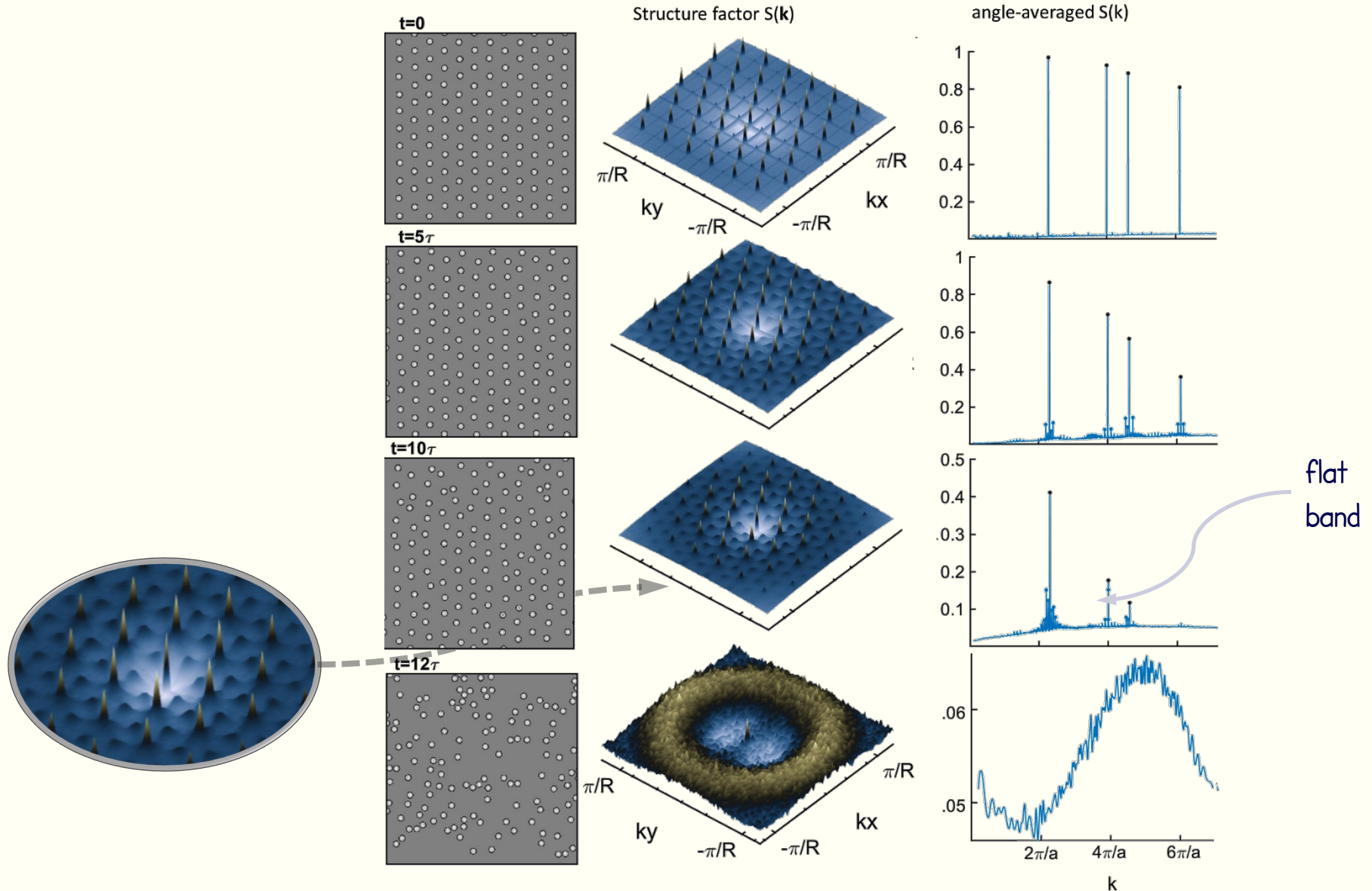
$$\omega_{\mathbf{k}} \sim \pm k^3 \cos 3\phi$$

A flat band of long-wavelength slow modes with vanishing velocity and curvature

$$\frac{\omega_{\mathbf{k}}}{k} = \partial_{\mathbf{k}} \omega_{\mathbf{k}} = \partial_{\mathbf{k}}^2 \omega_{\mathbf{k}} = 0$$



# Another path to disorder: melting of hexagonal crystals induced by flat band modes



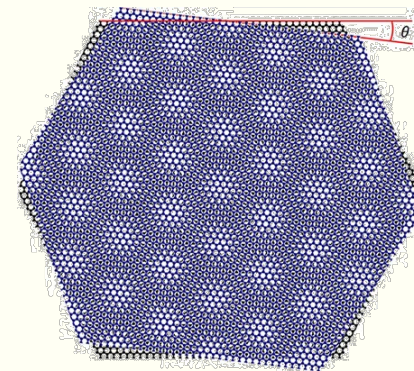
In flat bands slowing down of excitations leads to strong correlations

Structure factor  $S(k)$



**FLAT BANDS** are associated with high- $T_c$  superconductivity and topological insulators .

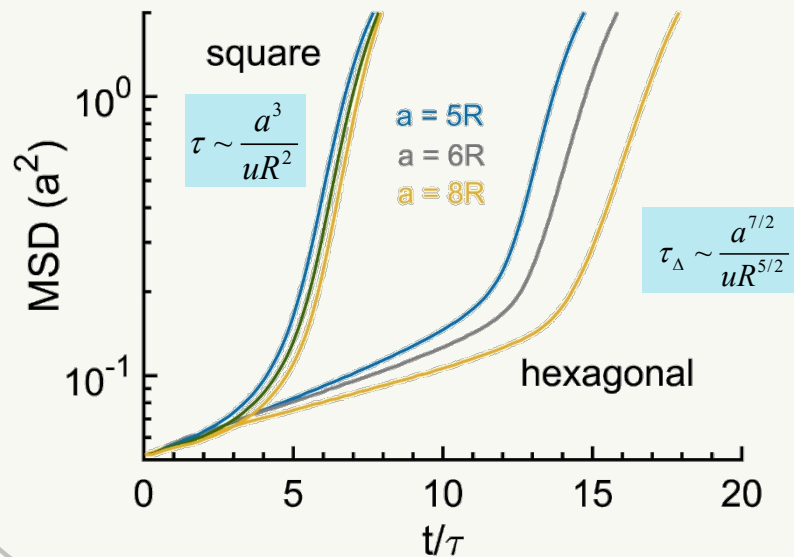
**GRAPHENE** bilayers exhibit flat bands and pairing when twisted at magic angle or buckled.



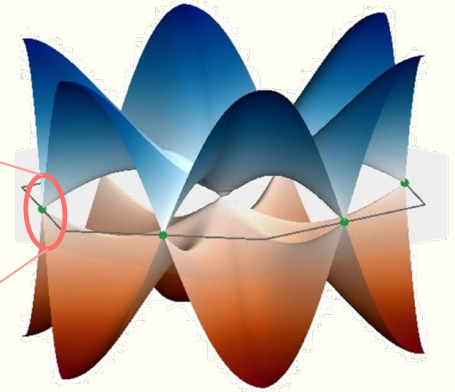
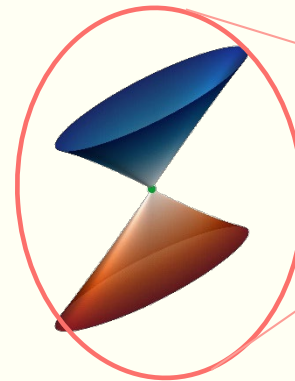
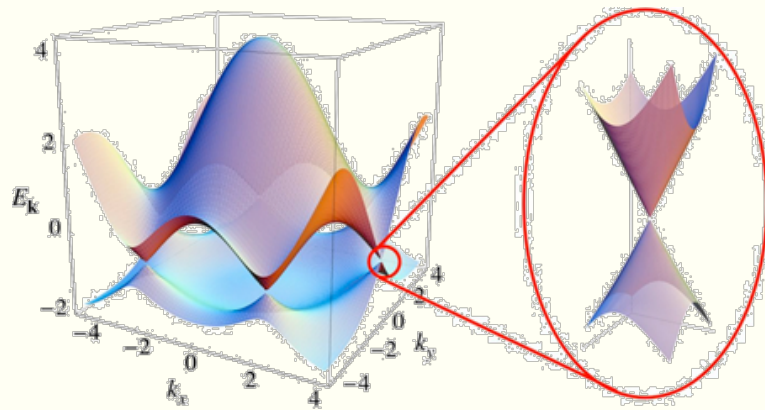


## Flat-band-induced melting is a much sharper and collective non-eq. transition

Manifested also in **scaling** of disorder parameter



## Certain similarities and mostly dissimilarities to Graphene



[Castro Neto et al. 2011]

|                      | Graphene                                             | Hydrodynamic crystal                                                   |
|----------------------|------------------------------------------------------|------------------------------------------------------------------------|
| Quasiparticle origin | symmetry of the <b>honeycomb crystal</b>             | <b>intrinsic parity symmetry of hydrodynamics</b>                      |
| Spin                 | pseudo-spin of electrons                             | polarization of phonons                                                |
| Singularities        | Dirac cones at the corners;<br>van Hove at boundary. | Dirac points at the corners;<br>van Hove <b>inside</b> Brillouin zone. |
| Flat bands           | by twisting bilayers at a magic angle                | <b>matching crystal and interaction symmetries</b>                     |

## Non-Hermitian systems exhibit hallmarks unmatched by Hermitian counterparts

### NON-HERMITICITY is *everywhere*:

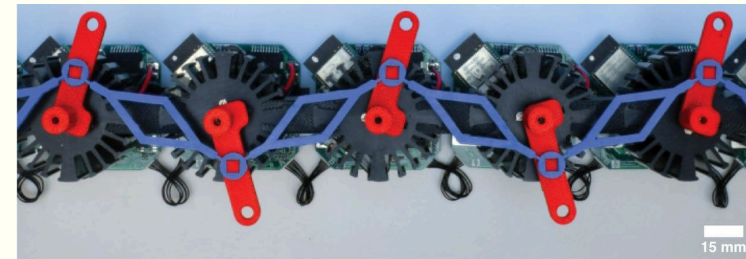
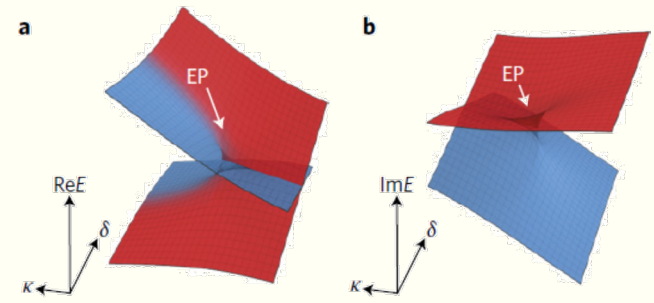
- most systems are **OPEN** and **Dissipative**.
- Any **EFFECTIVE**, coarse-grained system is open.

### Exotic TOPOLOGICAL PHENOMENA:

- Skewed spectral bands, exceptional points, fractional charges, PT symmetry breaking ....

Photonic, phononic, electronic, atomic lattices...  
recently in *active matter* (e.g. odd viscosity and elasticity)

$$H \neq H^\dagger$$



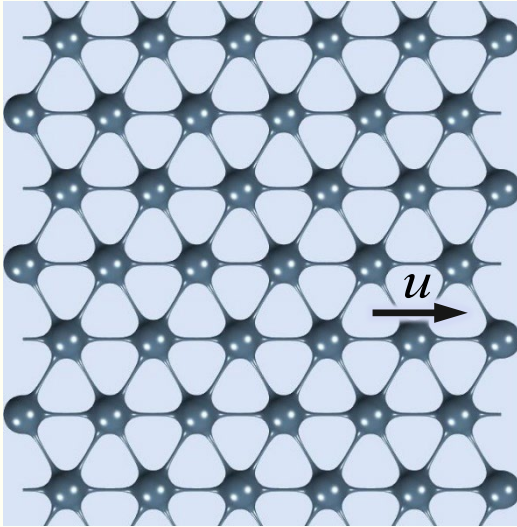
*Can we observe exotic topological phenomena in more common settings?*

...we are immersed in a dissipative, non-Hermitian world, and living systems are immanently open



## SIMPLE ANSWER:

### The interplay of elasticity and hydrodynamics induces non-Hermiticity



hydrodynamic dipoles

$$\mathbf{f}^{\text{hyd}}(\mathbf{r}) \sim \gamma u \frac{R^2}{r^2} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$$

$$\mathbf{f}(-\mathbf{r}) = \mathbf{f}(\mathbf{r})$$

+ **springs** (central conservative forces)

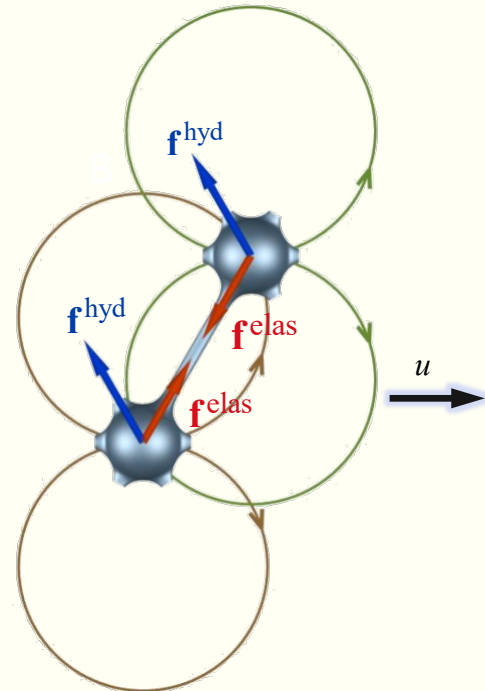
$$\mathbf{f}^{\text{elas}}(\mathbf{r}) \sim \kappa \Delta r \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

$$\mathbf{f}(-\mathbf{r}) = -\mathbf{f}(\mathbf{r})$$

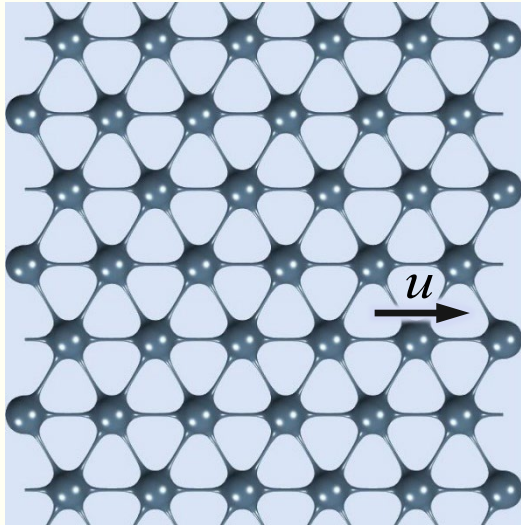
The "hydro-elastic" number

$$\mathcal{E} = \frac{1/\tau_{\text{elas}}}{1/\tau_{\text{hyd}}} = \frac{\kappa a^3}{\gamma u R^2}$$

$\kappa$  - spring constant  
 $\gamma$  - drag coeff.  
 $u$  - crystal velocity  
 $a$  - lattice constant  
 $R$  - particle size



# Hydrodynamic and elasticity have opposite parity and Hermiticity



- Equations of motion 
$$\gamma \dot{\mathbf{r}}_i = \mathbf{F} + \sum_j \left( \mathbf{f}_{ij}^{\text{hyd}} + \mathbf{f}_{ij}^{\text{elas}} \right).$$

- Normal modes: 
$$\delta \mathbf{r}_i = \boldsymbol{\psi}_{\mathbf{k}} e^{i(\mathbf{k} \cdot \bar{\mathbf{r}}_i - \omega t)}.$$

- $\boldsymbol{\psi}_{\mathbf{k}}$  is eigenvector of 
$$\omega_{\mathbf{k}} \boldsymbol{\psi}_{\mathbf{k}} = \mathbf{H} \boldsymbol{\psi}_{\mathbf{k}}$$

with the hydro-elastic operator

$$\mathbf{H} = \mathbf{H}_{\text{hyd}} + \varepsilon \mathbf{H}_{\text{elas}}$$

$$\begin{pmatrix} \Omega_x \\ \Omega_y \end{pmatrix} \sim \sum \frac{e^{i\mathbf{k} \cdot \bar{\mathbf{r}}_j}}{\bar{\mathbf{r}}_j^3} \begin{pmatrix} \cos 3\theta_j \\ \sin 3\theta_j \end{pmatrix}$$

$$\mathbf{H}_{\text{hyd}} = \boldsymbol{\Omega} \cdot \boldsymbol{\sigma}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_l \end{pmatrix} \sim \sum e^{i\mathbf{k} \cdot \bar{\mathbf{r}}_j} \begin{pmatrix} \cos 2\theta_j \\ \sin 2\theta_j \\ 1 \end{pmatrix}$$

$$\mathbf{H}_{\text{elas}} = -i(\boldsymbol{\omega} \cdot \boldsymbol{\sigma} + \omega_l \cdot \mathbf{1})$$

- Hydrodynamics is **Hermitian** and odd under parity;  
Elasticity is **skew-Hermitian** and **even**.

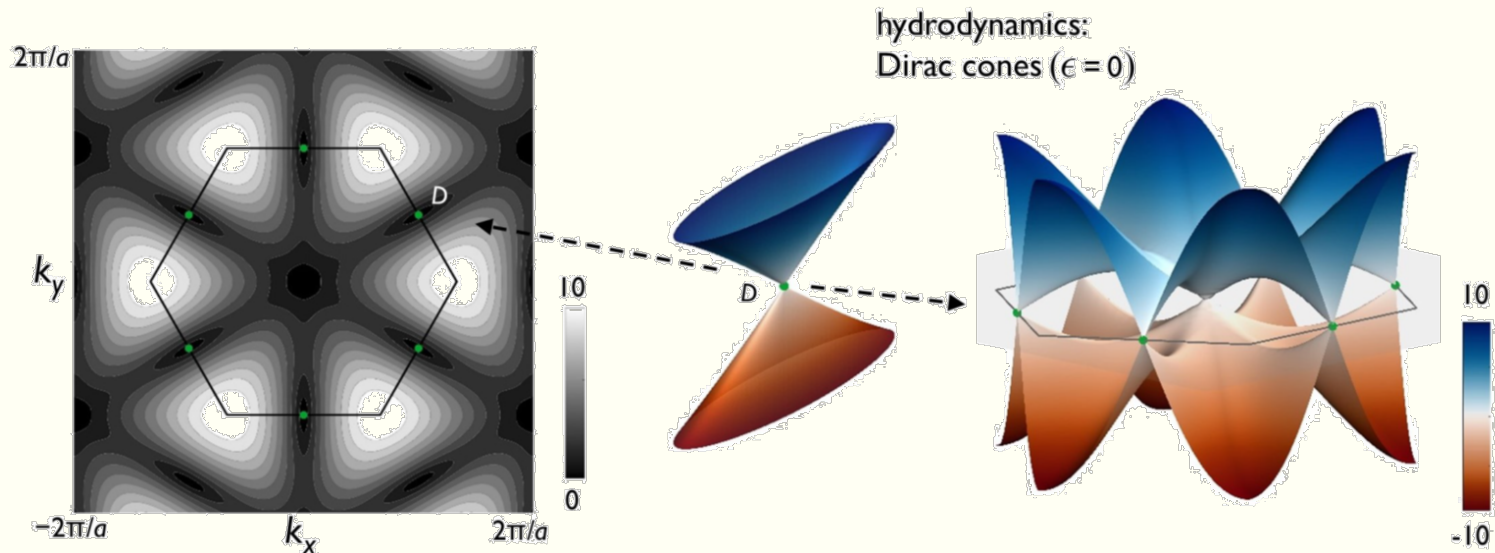
$$\mathbf{H}_{\text{hyd}} = \mathbf{H}_{\text{hyd}}^\dagger, \quad \mathbf{H}_{\text{hyd}}(-\mathbf{k}) = -\mathbf{H}_{\text{hyd}}(\mathbf{k})$$

$$\mathbf{H}_{\text{elas}} = -\mathbf{H}_{\text{elas}}^\dagger, \quad \mathbf{H}_{\text{elas}}(-\mathbf{k}) = \mathbf{H}_{\text{elas}}(\mathbf{k})$$



Questions ?

# The purely hydrodynamic system shows real phonon spectrum with Dirac cones



The two bands of the spectrum

$$\omega_{\pm} = i\epsilon\omega_1 + \sqrt{\nu_+\nu_-}$$

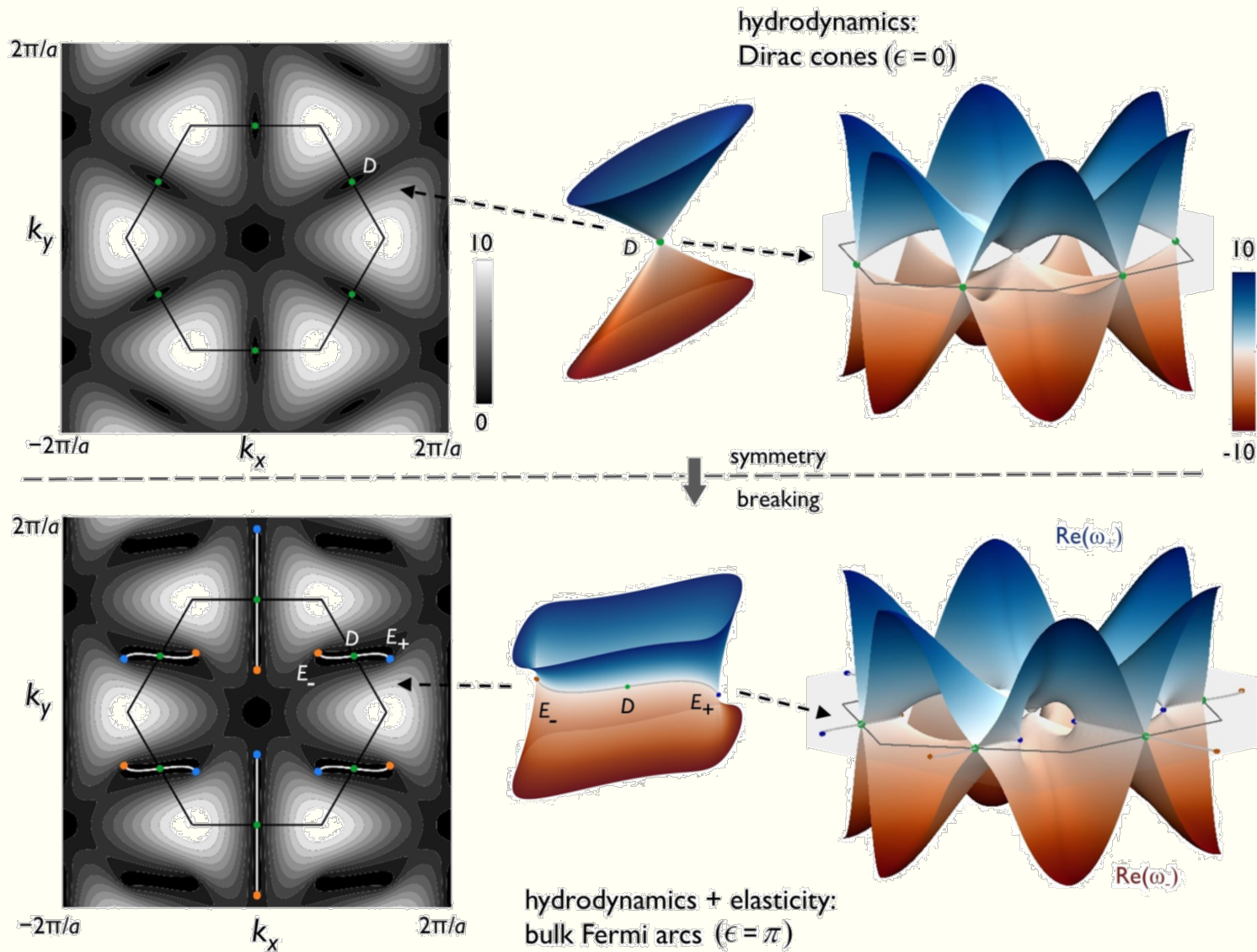
$$\nu_{\pm} = (\Omega_x - i\epsilon\omega_x) \pm (\Omega_y - i\epsilon\omega_y)$$

Polarization eigenvectors

$$\Psi_{\mathbf{k}} \sim \begin{pmatrix} \sqrt{\nu_{\pm}} \\ \sqrt{\nu_{\mp}} \end{pmatrix}$$



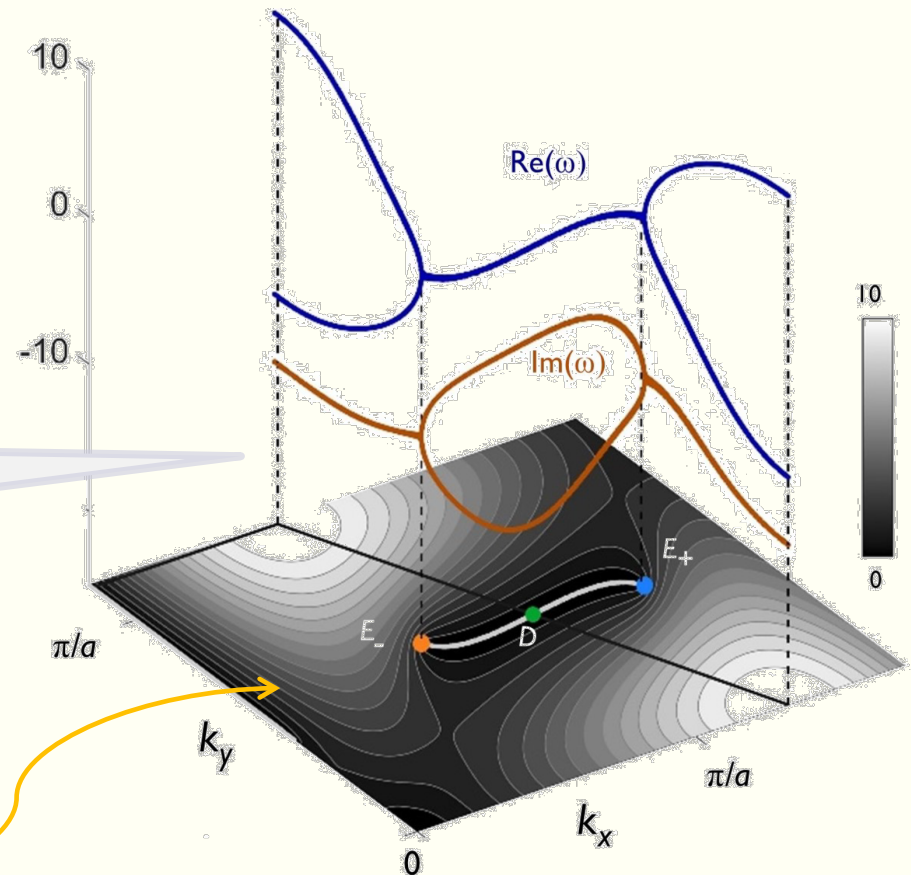
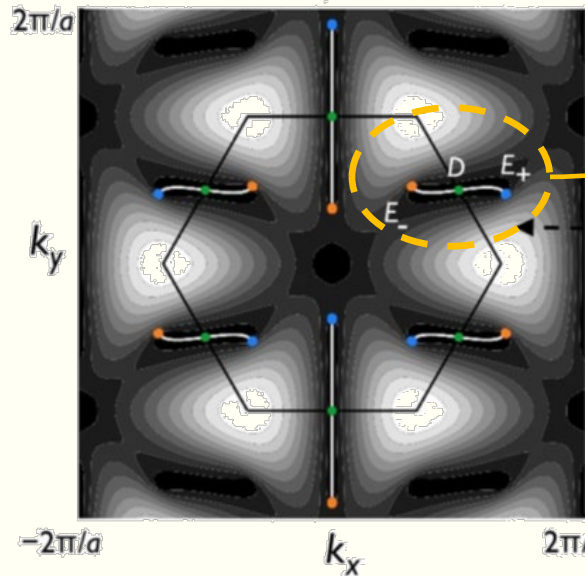
# Elasticity breaks the symmetry and splits the Dirac cones into Bulk Fermi Arcs





**Bulk Fermi\* Arcs** are open-ended iso-frequency contours joining a pair of exceptional points

**BULK FERMION\* ARCS** manifest non-Hermiticity in the *bulk* spectrum, unlike *surface* Fermi arcs induced by Weyl points in 3D Hermitian systems



At the **exceptional points**:

- Eigenvalues **and** eigenvectors coalesce.
- Bands are purely dissipative (real part = 0)
- **Square-root singularity** of the Riemann surface.



\* No electrons involved (as in photonic crystals)

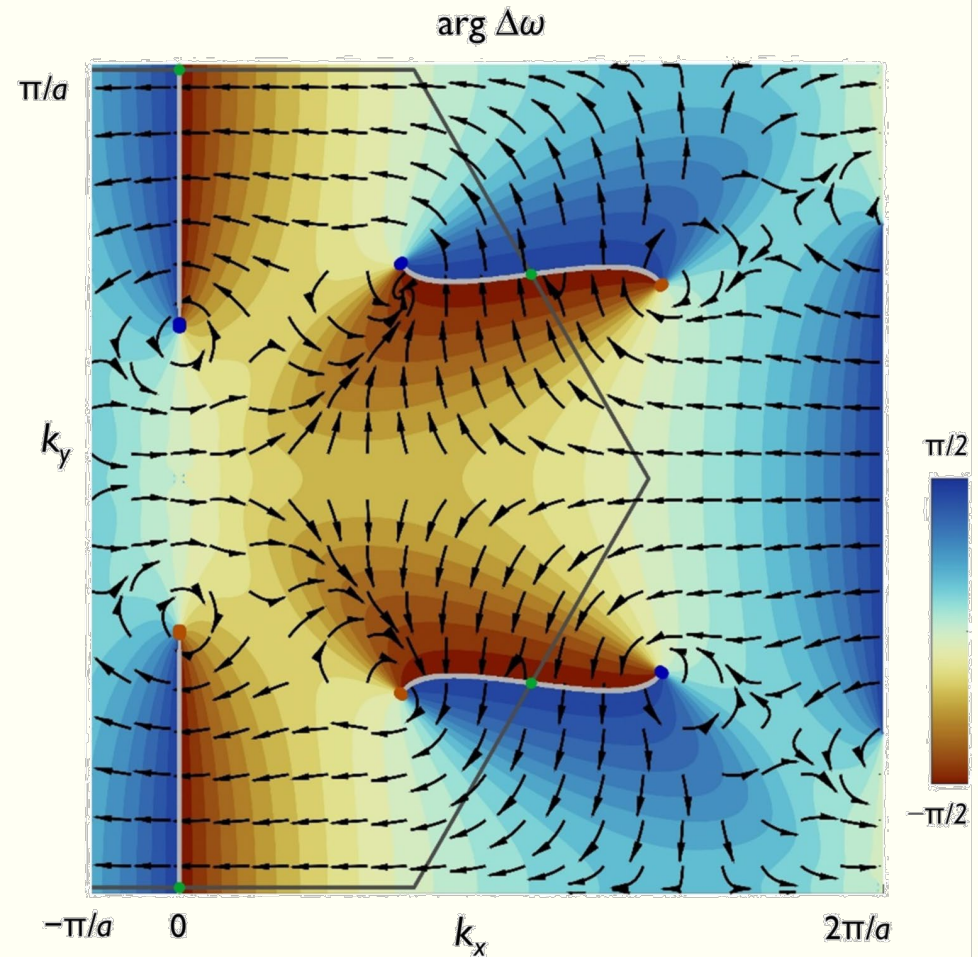
## The exceptional points are topological defects with half-integer charges

Charges are found from the Berry phase

$$\nu = -\frac{i}{2\pi} \oint d\mathbf{k} \cdot \boldsymbol{\psi}_{\mathbf{k}}^* \nabla_{\mathbf{k}} \boldsymbol{\psi}_{\mathbf{k}} = \pm \frac{1}{2}$$

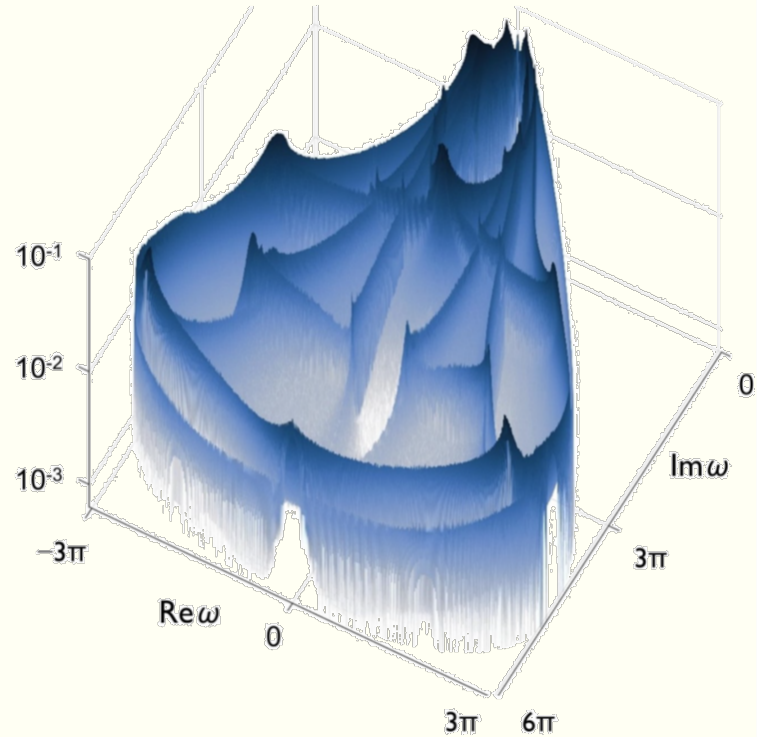
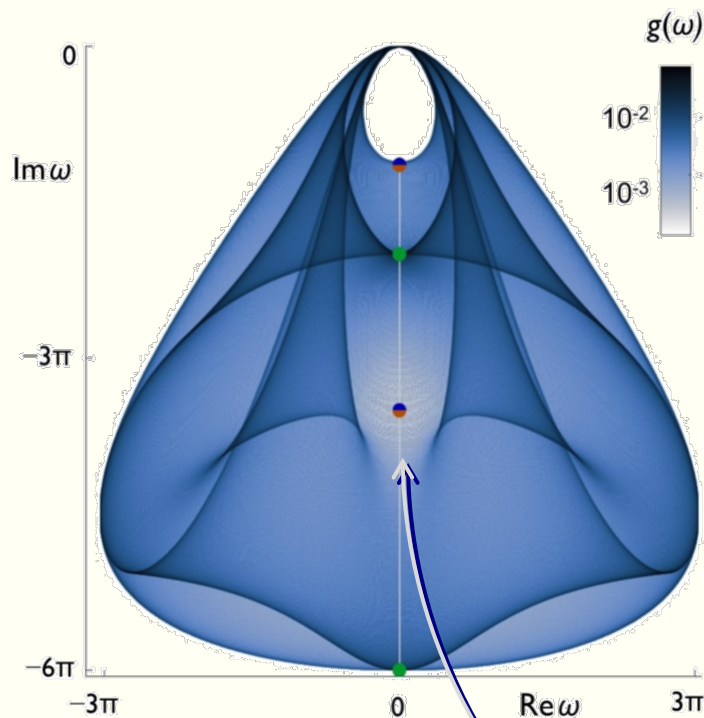
or other winding numbers, such as the  
**vorticity of the band gap:**

$$\nu = -\frac{1}{2\pi} \oint d\mathbf{k} \cdot \nabla_{\mathbf{k}} \arg(\omega_+ - \omega_-) = \pm \frac{1}{2}$$



The density of states show extended van Hove 'ridges' reflecting the topology

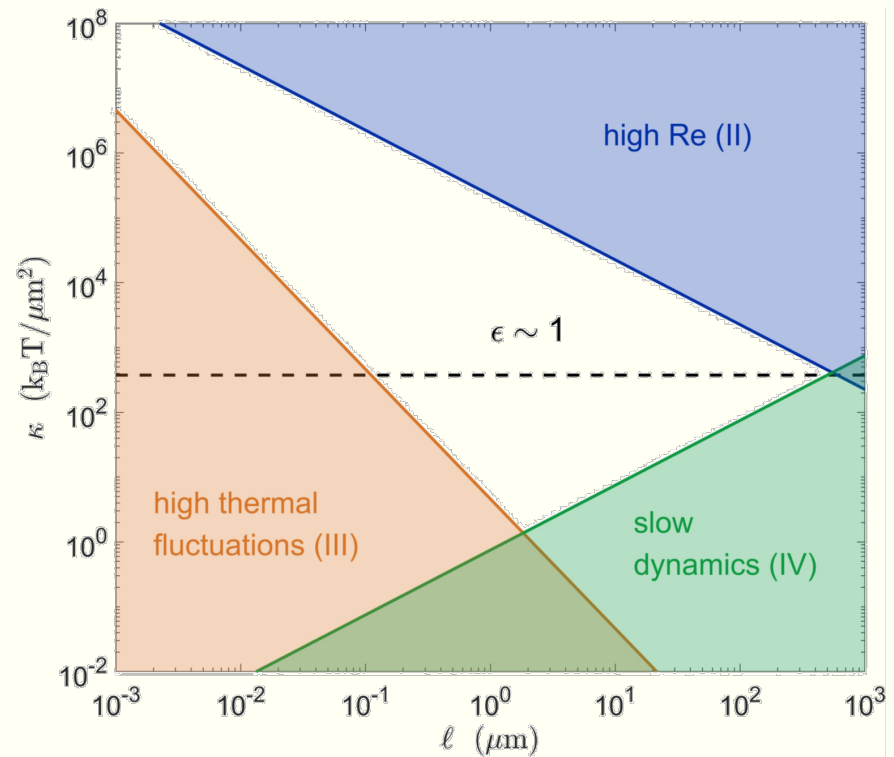
$$g(\omega) = \left( \frac{a}{2\pi} \right)^2 \int d^2\mathbf{k} \delta(\omega - \omega_{\mathbf{k}})$$



Level repulsion shows in low density of states around the exceptional points.

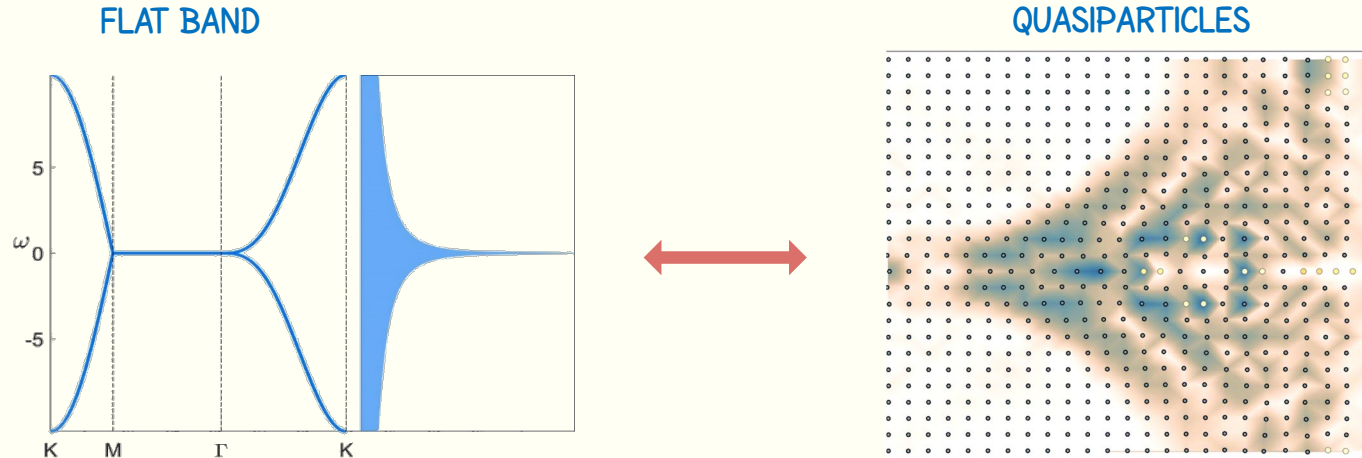
Proposal : Exceptional topology is **GENERIC** and should be observed in various soft matter systems subject to viscous flow

- **Potential systems**: colloidal crystals connected by DNA springs or effective springs induced by magnetic repulsion...
- Also at the nanoscale: lattices made of DNA, protein or patchy nano-particles.
- Periodic lattices and low-Reynolds flows are **omnipresent** in biology and biotechnology.



*Are non-Hermitian topological phenomena also widespread in these realms?*

- Basic notions of quantum solid theory - QUASIPARTICLES and BAND THEORY - provide insight into the many-body dynamics of a classical dissipative system

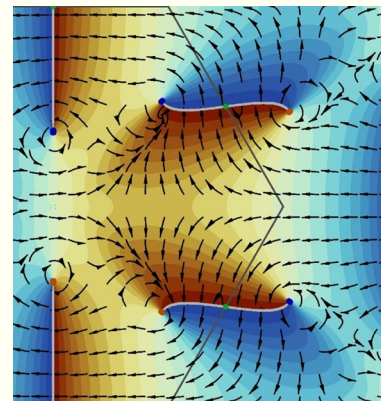


- EXOTIC TOPOLOGY is common in ordinary soft matter (in flow)

### THANKS

Imran SAEED, Hyuk Kyu PAK (IBS)

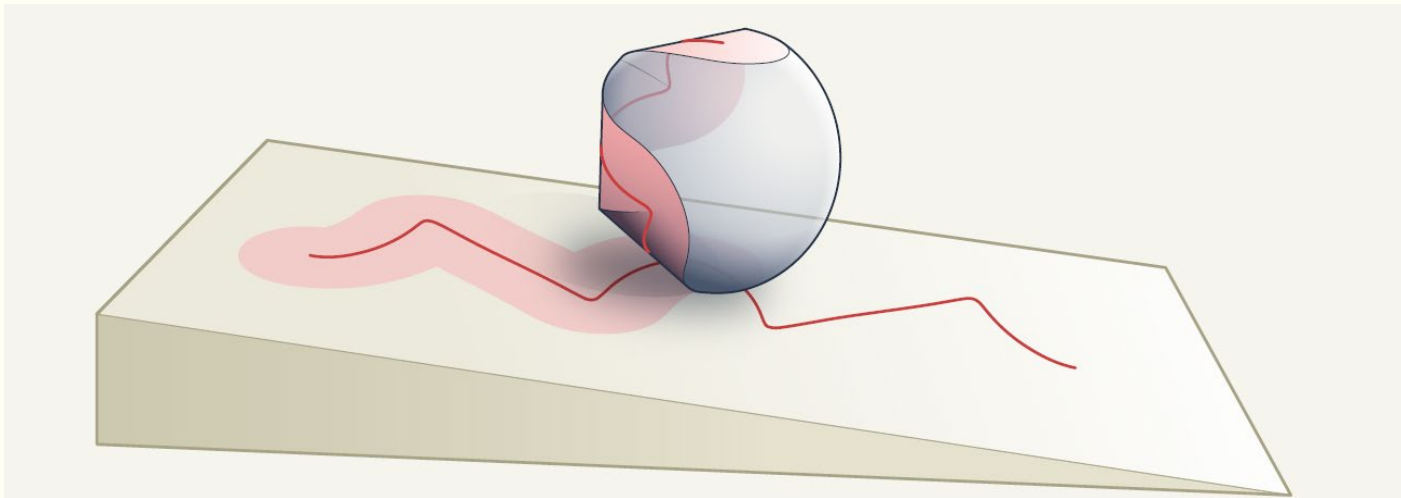
Tsevi BEATUS (HUJI), Roy BAR-ZIV (Weizmann)





## Other analogies to QM in daily life systems

Rolling along an infinite periodic path reveals basic properties of the rotation group  $SO(3)$



**Sobolev** et al. Nature 2023; **Eckmann** et al. AMS Notices 2024;

**Eckmann** et al. Walks in Rotation Spaces Return Home When Doubled and Scaled. arXiv:2502.14367

