

Feb 2022, UCSB

THE USEFULNESS OF QUANTUM CONCEPTS IN SOFT MATTER
QUASIPARTICLES, FLAT BANDS, AND EXOTIC
TOPOLOGY IN HYDRODYNAMIC MATTER



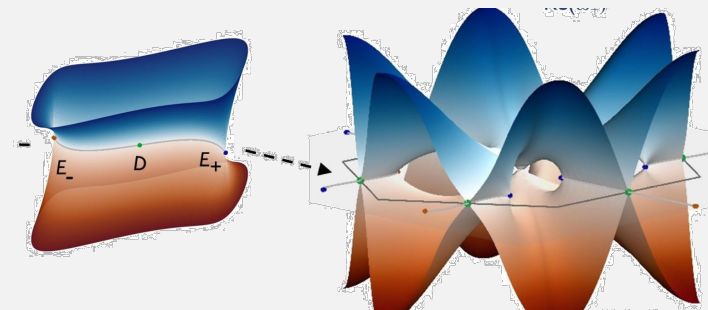
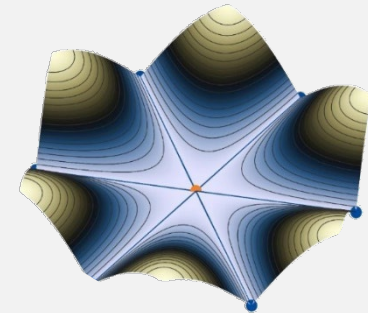
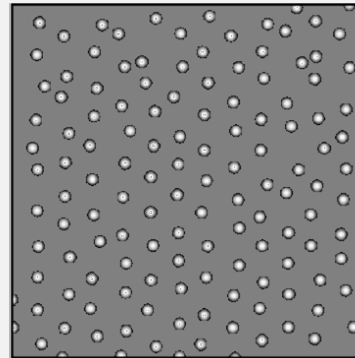
Imran Saeed & Hyuk Kyu Pak (IBS)

Collective phenomena in
classical dissipative systems
at room temperature



Collective phenomena in
quantum matter:
quasiparticles, flat band...

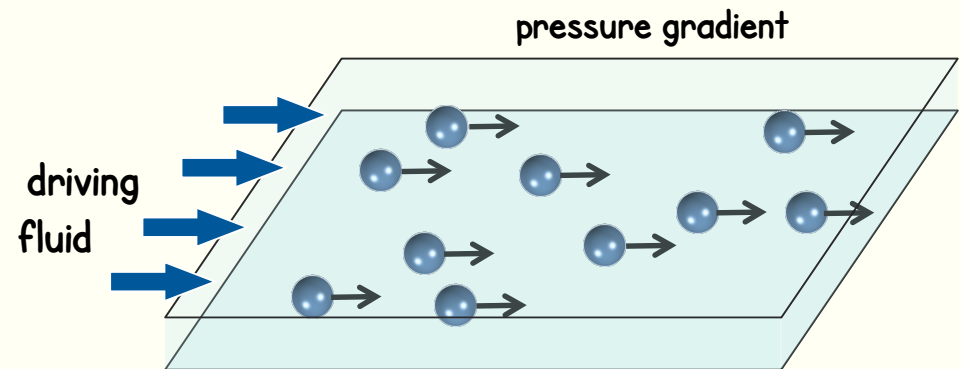
- Part 1 (Hermitian):
Non-equilibrium transitions in a
hydrodynamic crystal
- Part 2 (non-Hermitian):
Emergence of exceptional topology by
coupling elasticity and hydrodynamics.



Please ask questions during the talk

A CLASSICAL DISSIPATIVE MODEL SYSTEM:

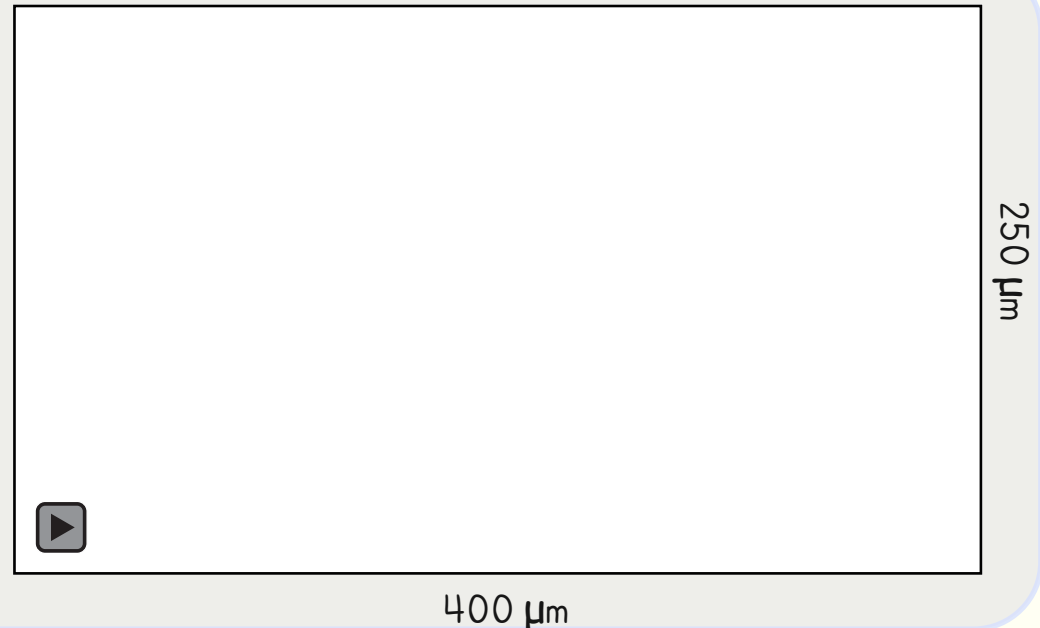
2D ensemble of micro-particles
driven in a thin sheet of fluid



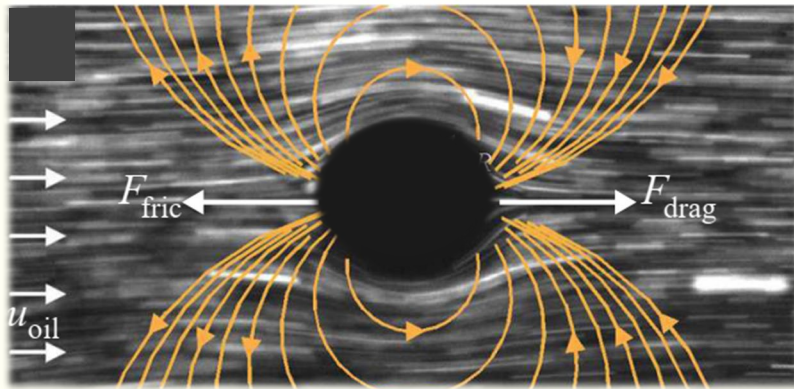
Microfluidic channel with driven colloids:

- **Dilute** system: 1.8% area density.
- System is **effectively 2D**: Diameter - 7 μm , channel height - 10 μm
- **Friction**: velocity **relative** to fluid: 100 $\mu\text{m/s}$

Saeed, Pak , TT (in review)



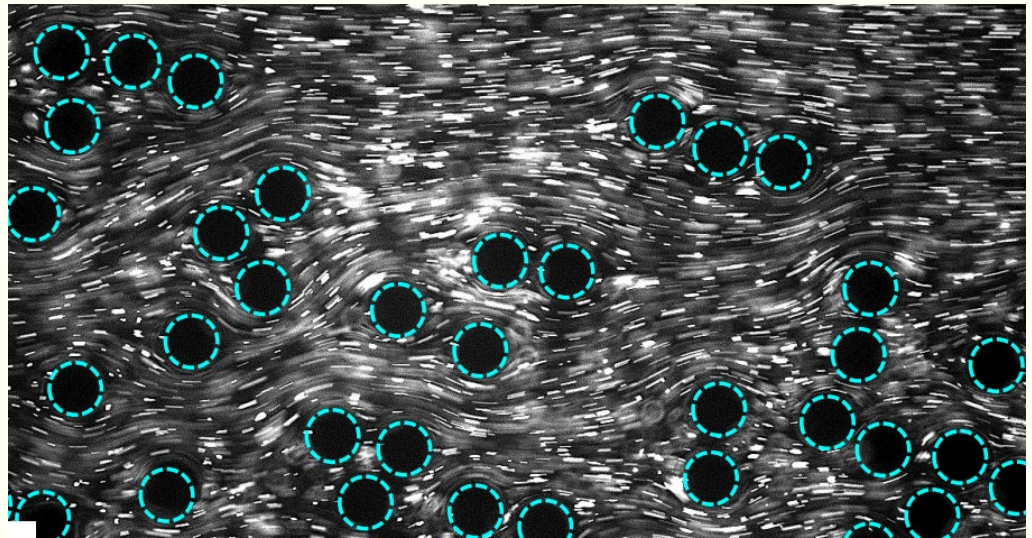
Particles driven in a quasi-2D geometry induce hydrodynamic dipoles



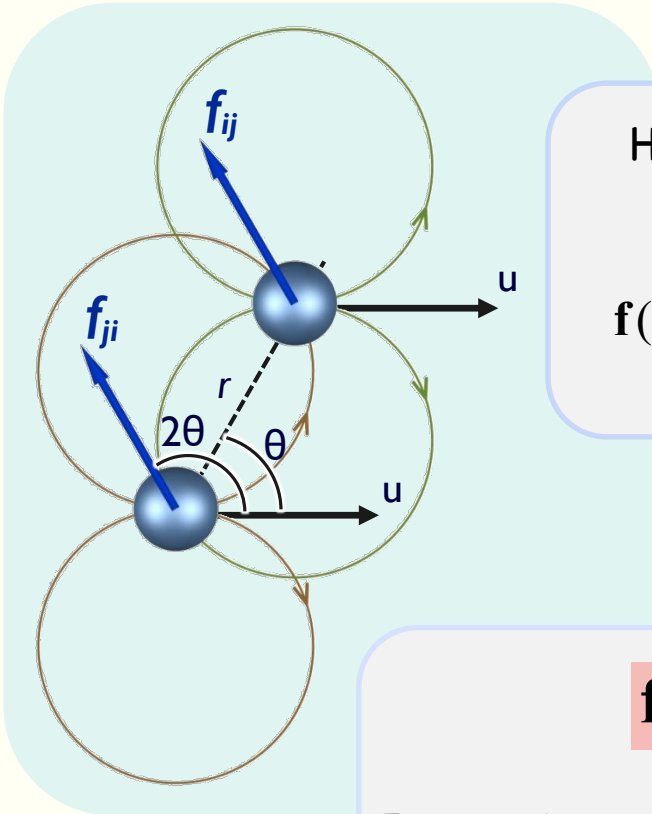
- Reynolds number: $10^{-4} \rightarrow$ forget inertia.
- Peclet number $10^4 \rightarrow$ also thermal forces.

$$\mathbf{F}_{fric} = \gamma \dot{\mathbf{r}}_i = \mathbf{F}_{drag}$$

Particles interact by perturbing streamlines and thereby drag forces.



Twofold symmetry of the forces leads to invariance under parity



Hydrodynamic dipole

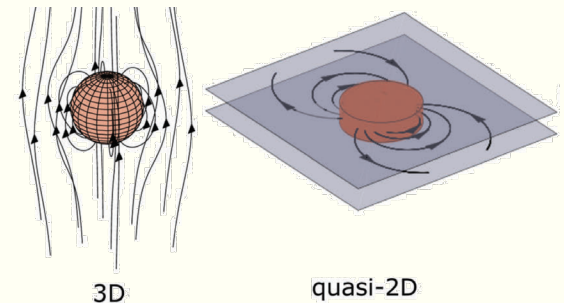
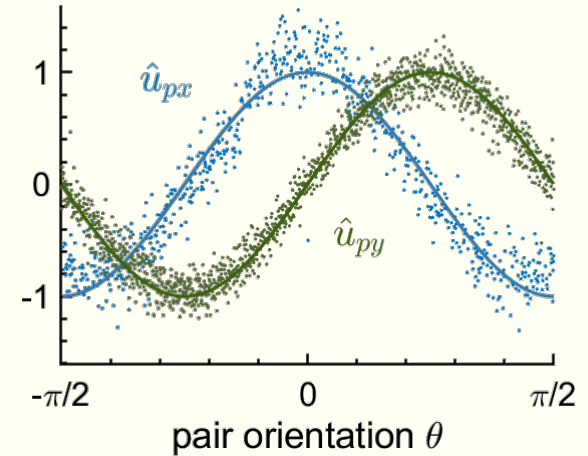
$$\mathbf{f}(\mathbf{r}) \sim \frac{1}{r^2} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$$

$$\mathbf{f}(-\mathbf{r}) = \mathbf{f}(\mathbf{r})$$

Two particles exert on each other **EQUAL FORCES**.

[Unlike Newton's 3rd law: $\mathbf{f}(-\mathbf{r}) = -\mathbf{f}(\mathbf{r})$]

Measured pair velocity



$$v \sim \frac{R}{r} + \left(\frac{R}{r}\right)^3$$

Momentum
monopole

$$v \sim \left(\frac{R}{r}\right)^2$$

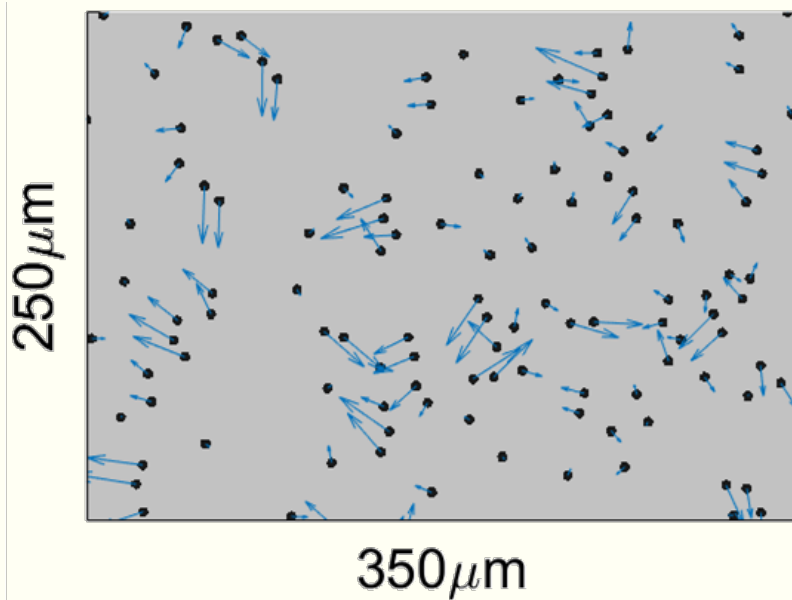
Mass dipole



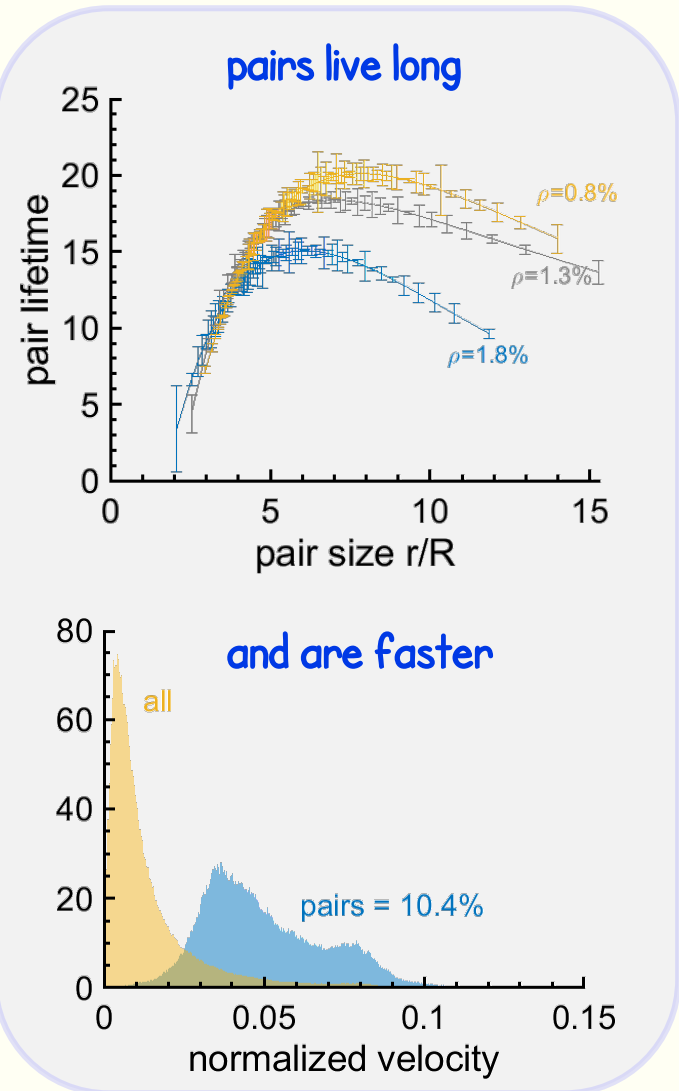
This is the origin of all physical phenomena we will discuss

Weakly-interacting pairs are stable between collisions

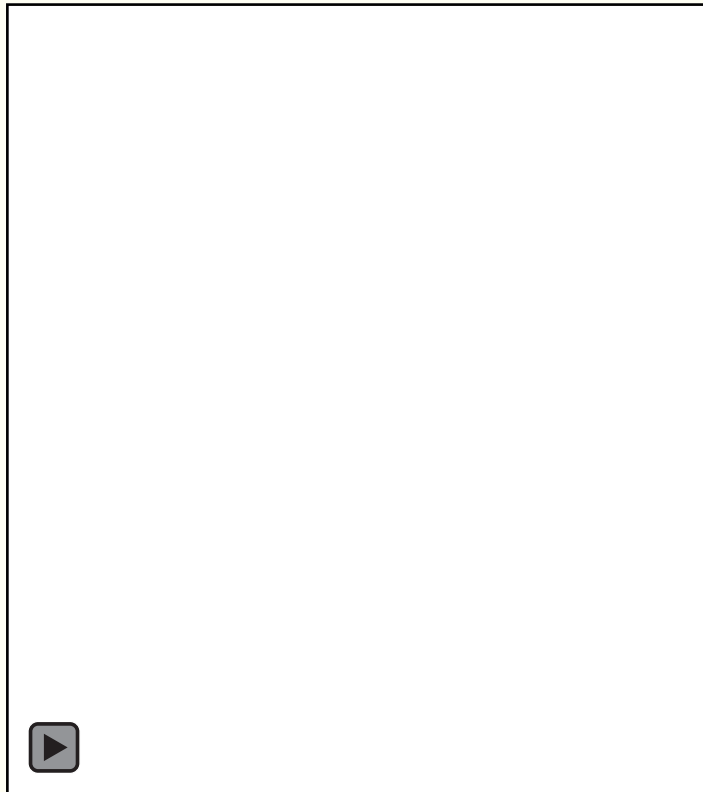
(> 10^5 measured particles)



Are pairs elementary particle-like excitations?



Long-range forces excite phonons in the hydrodynamic crystal



Simulation 51 x 51 particles



Please ask questions

Equations of motion $\gamma \dot{\mathbf{r}}_i = \sum_j \mathbf{f}_{ij} + \mathbf{F}$

Uniform steady-state motion at $\dot{\mathbf{r}}_i = \mathbf{F} / \gamma$

Normal modes - "phonons" - around steady state
are plane waves $\delta \mathbf{r}_i = \boldsymbol{\psi}_{\mathbf{k}} e^{i(\mathbf{k} \cdot \bar{\mathbf{r}}_i - \omega t)}$

2D polarization $\boldsymbol{\psi}_{\mathbf{k}}$ is an eigenvector
of a "Schrödinger" equation

$$\omega_{\mathbf{k}} \boldsymbol{\psi}_{\mathbf{k}} = H \boldsymbol{\psi}_{\mathbf{k}}$$

with the **TRIPOLAR** "spring constant"

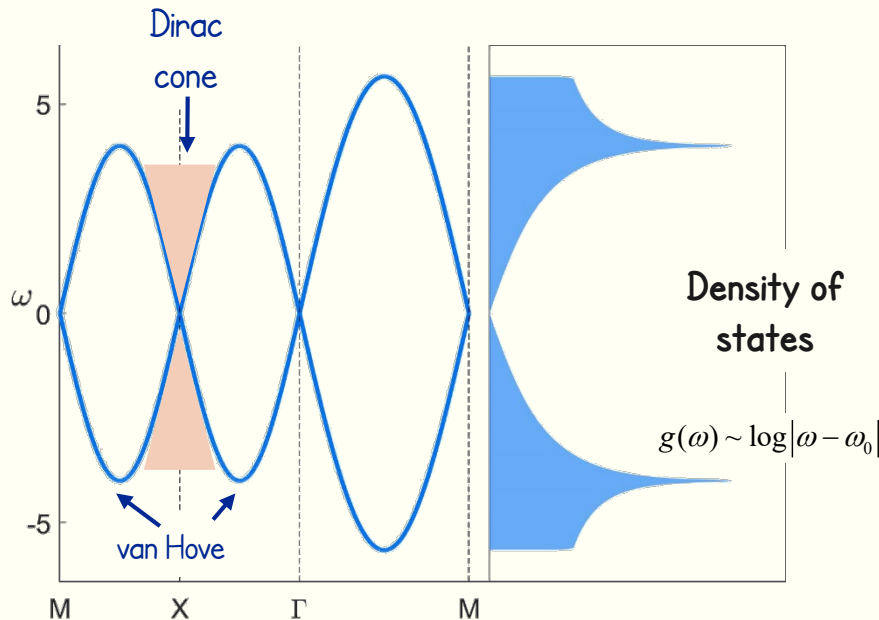
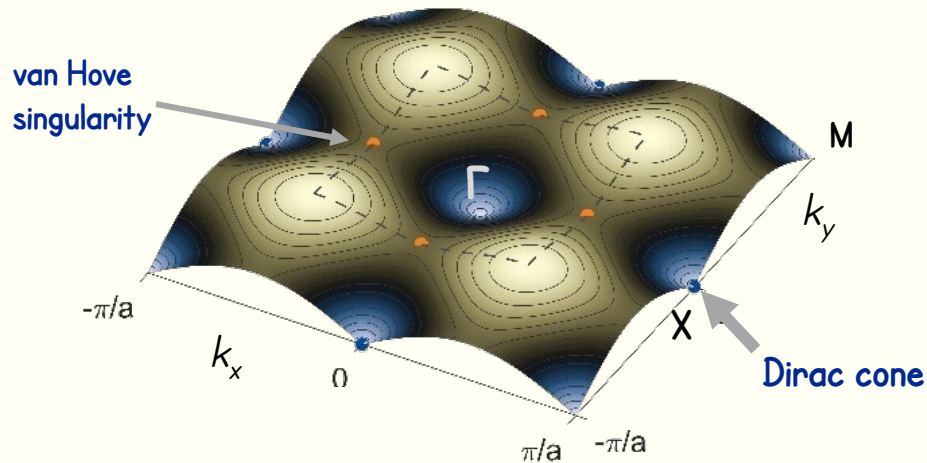
$$H = \boldsymbol{\Omega} \cdot \boldsymbol{\sigma},$$

$$\boldsymbol{\Omega} = \begin{pmatrix} \Omega_x \\ \Omega_y \end{pmatrix} \sim \sum_j \frac{e^{i\mathbf{k} \cdot \bar{\mathbf{r}}_j}}{\bar{\mathbf{r}}_j^3} \begin{pmatrix} \cos 3\theta_j \\ \sin 3\theta_j \end{pmatrix}.$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix}. \quad \text{Pauli's matrices}$$

The spectrum of hydrodynamic phonons hints for quasiparticles

1st Brillouin zone



The spectrum is purely real with propagating phonons

$$\omega_{\mathbf{k}} = \pm |\mathbf{\Omega}| = \pm \sqrt{\Omega_x^2 + \Omega_y^2}$$

In a square lattice with nearest neighbors

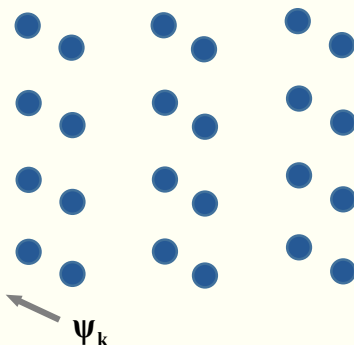
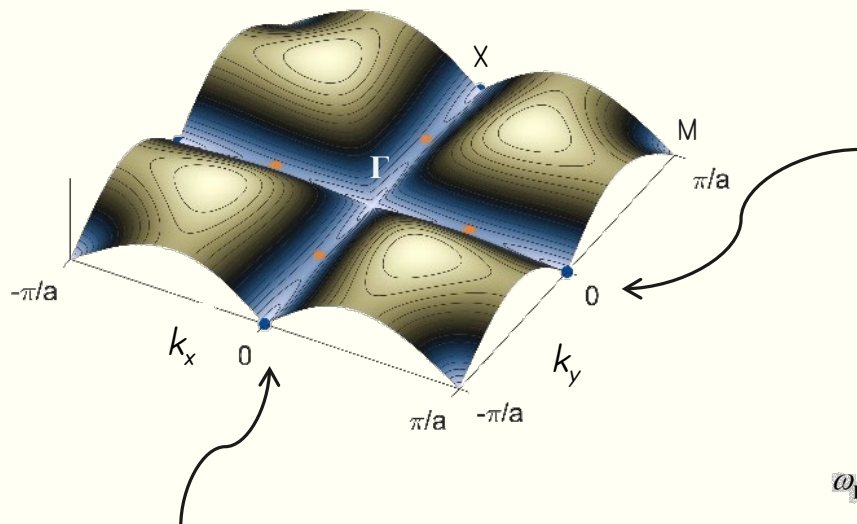
$$\omega_{\mathbf{k}} \sim \pm \sqrt{2 - \cos 2k_x a - \cos 2k_y a}$$

THREE HALLMARKS:

- **Dirac cones** (X).
- van Hove singularities *inside* Brillouin zone.
- **Smaller** primitive cell (period = $2a$).

Excitations at the Dirac points are pairing modes

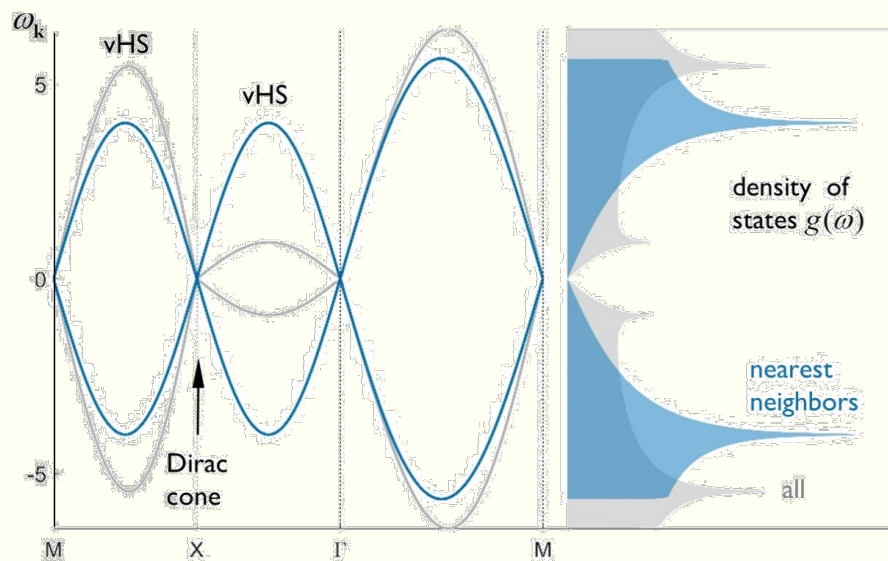
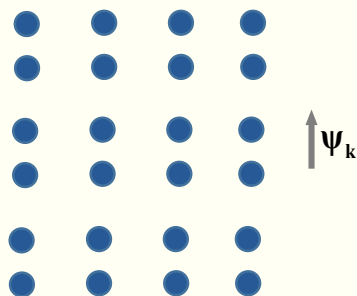
all neighbors



Dirac cone = acoustic modes of pairing lines

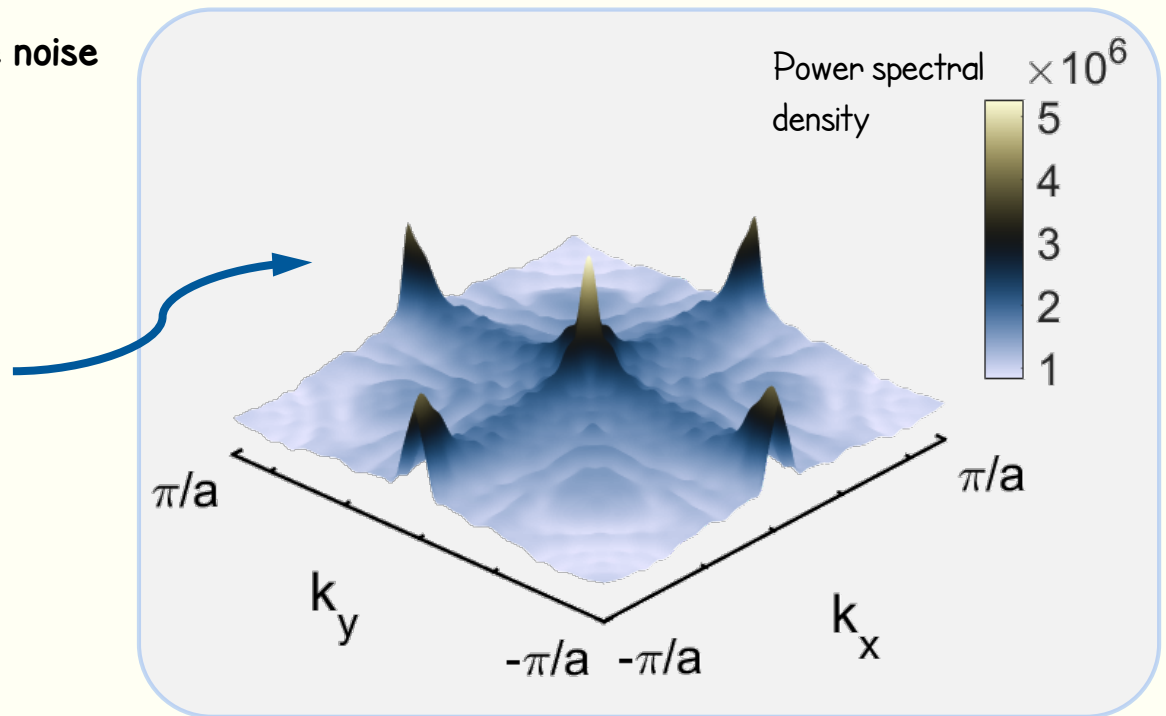
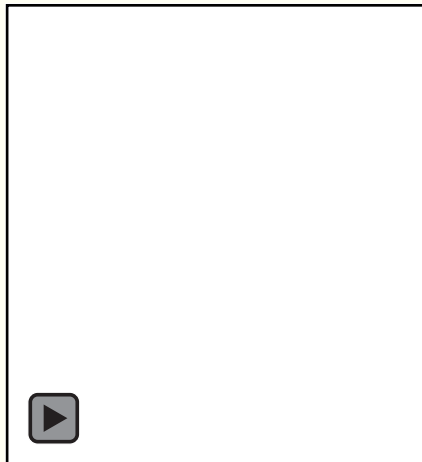
$$\omega \sim c \cdot (\mathbf{k} - \mathbf{k}_D)$$

pairing modes

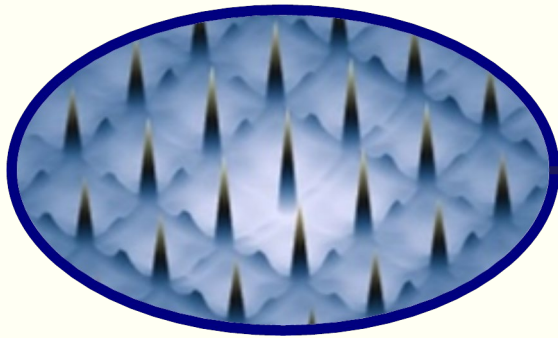


Pairing modes emerge via *nonlinear* mode coupling (phonon-phonon processes)

Simulation starting from white noise

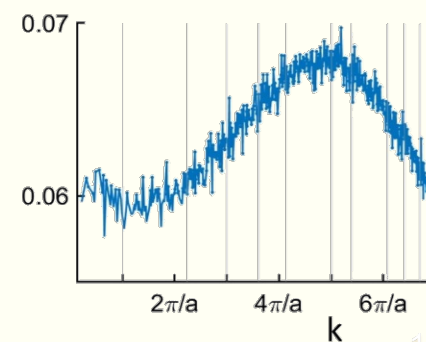
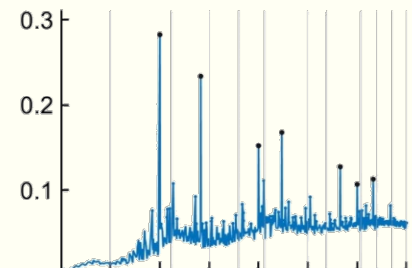
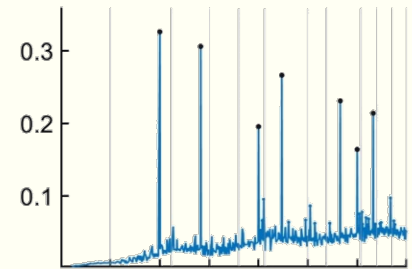
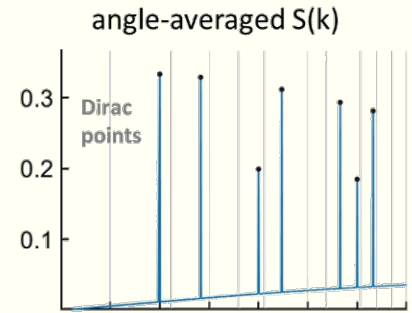
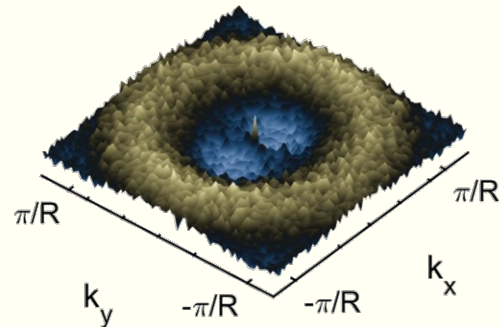
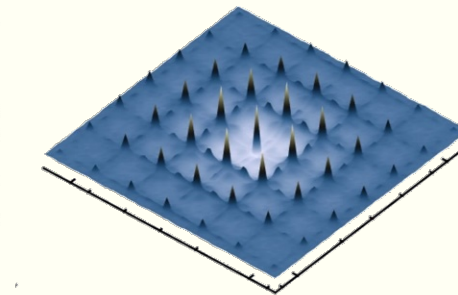
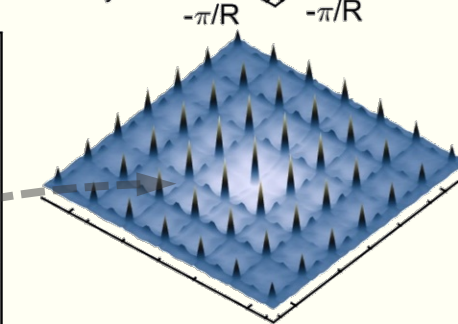
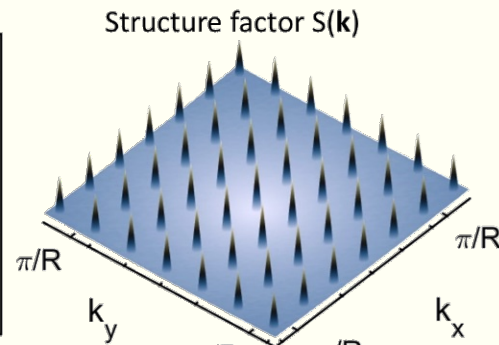
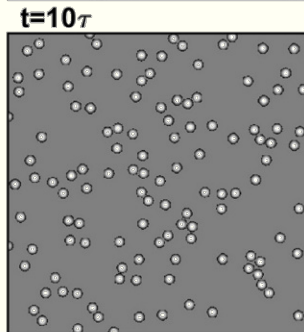
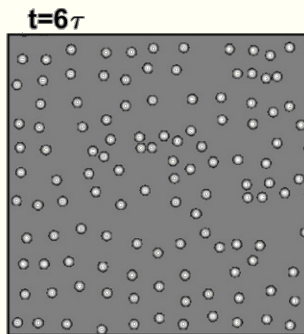
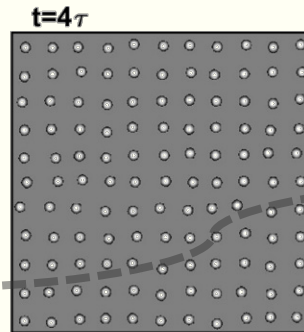
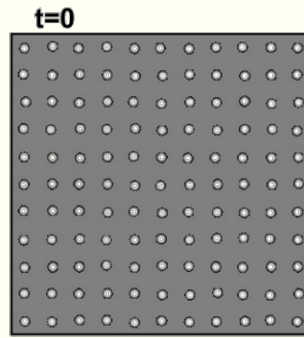


Pairs induce the melting of the hydrodynamic crystal



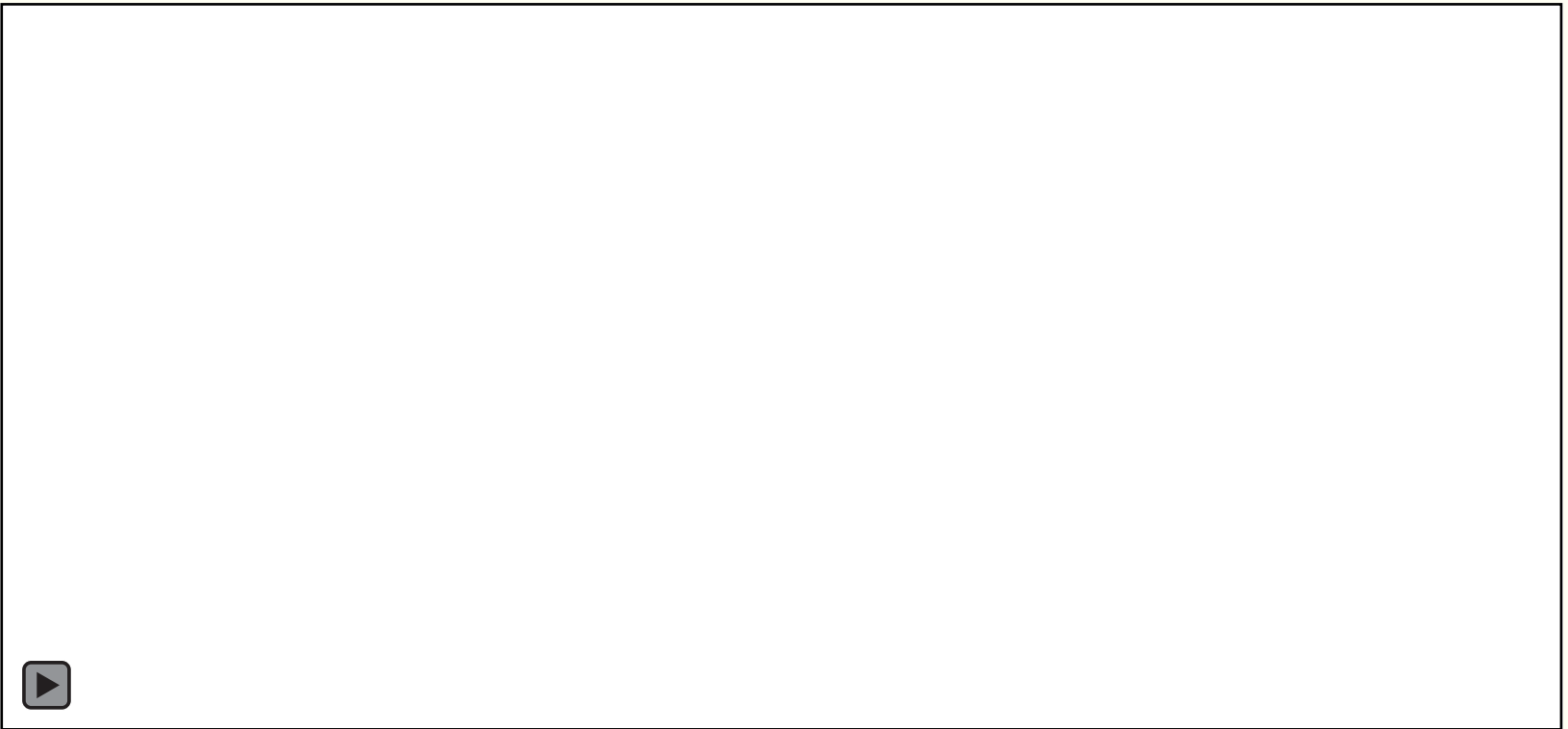
Natural timescale in crystal

$$\tau = \frac{a}{\text{sound speed}} \sim \frac{a^3}{uR^2}$$



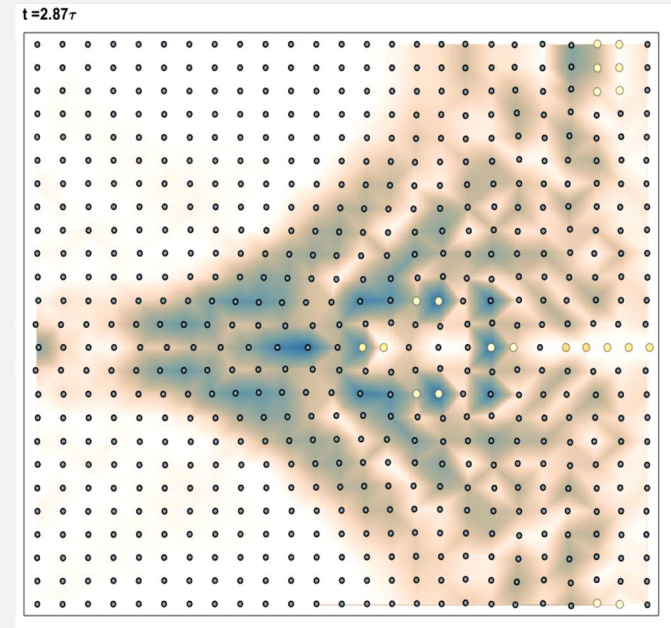
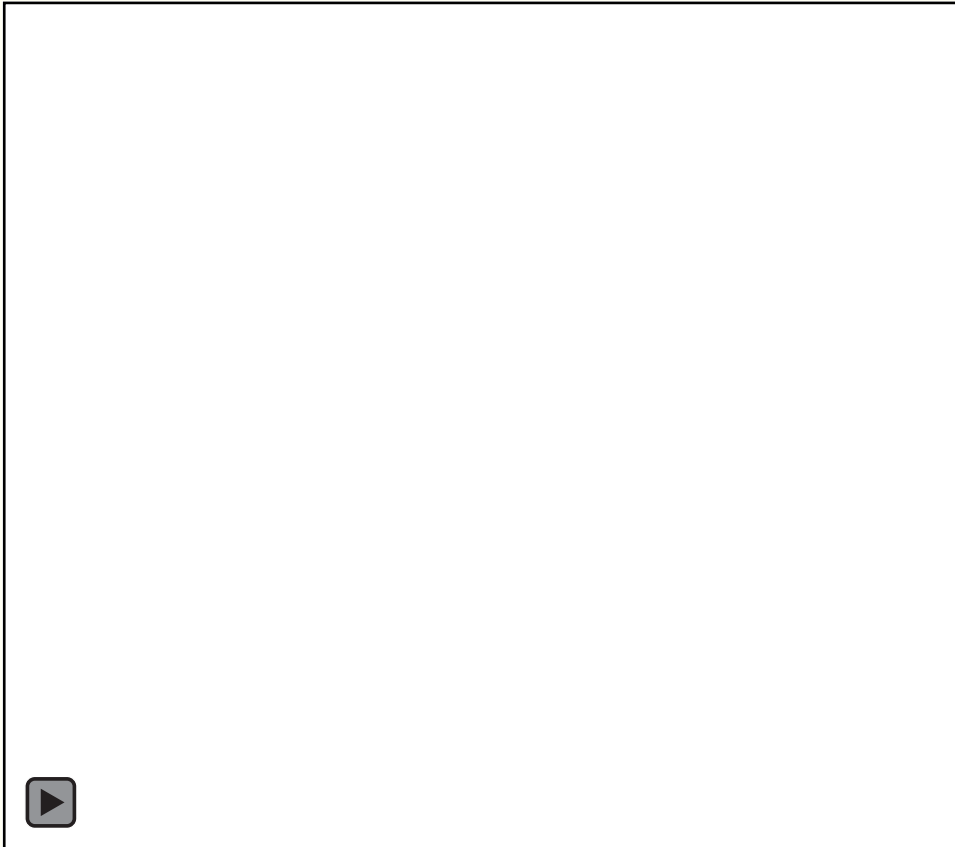
Questions?

Pairs induce the melting of the hydrodynamic crystal



Quasiparticles excite supersonic pairing avalanches, leading to melting

Localized perturbation ("Green function")



Mach cone of pairing trailing a quasiparticle.

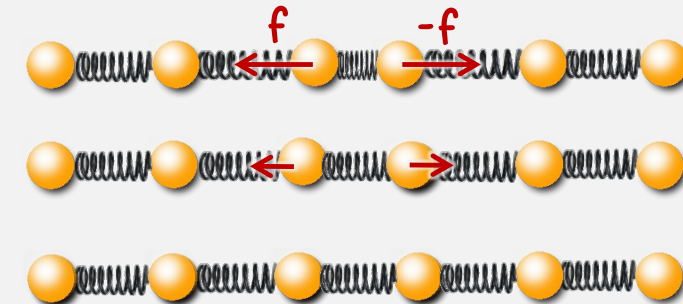
Mach # = 1.4 (shading indicates velocity)

$f(-r) = f(r)$ parity makes quasiparticles and phonons propagate freely

In equilibrium crystals (e.g. elastic):

force $\sim \nabla \text{strain} \sim \nabla^2 \text{displacement}$

$$\gamma \dot{u} = \kappa \nabla^2 u \Rightarrow \omega \sim i \sqrt{\frac{\kappa}{\gamma}} k^2$$



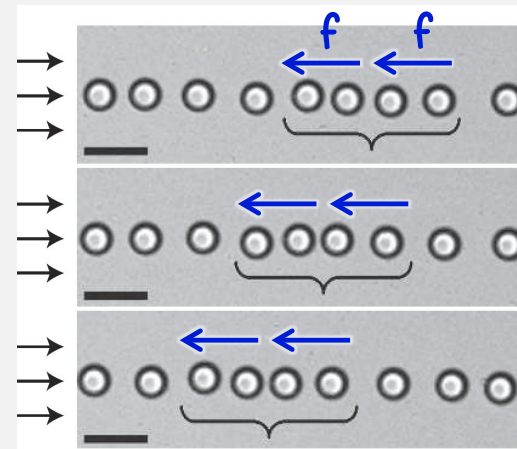
waves and disturbances diffuse and dissipate;
crystal relaxes to equilibrium

In hydrodynamic crystals:

force $\sim \text{strain} \sim \nabla \text{displacement}$

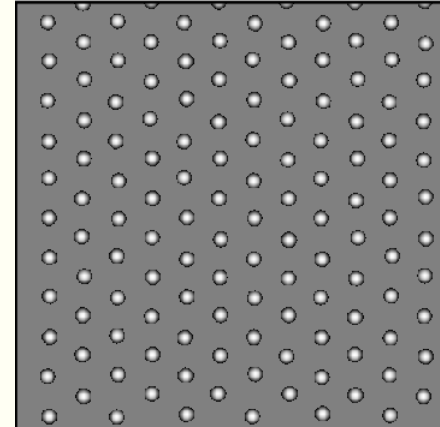
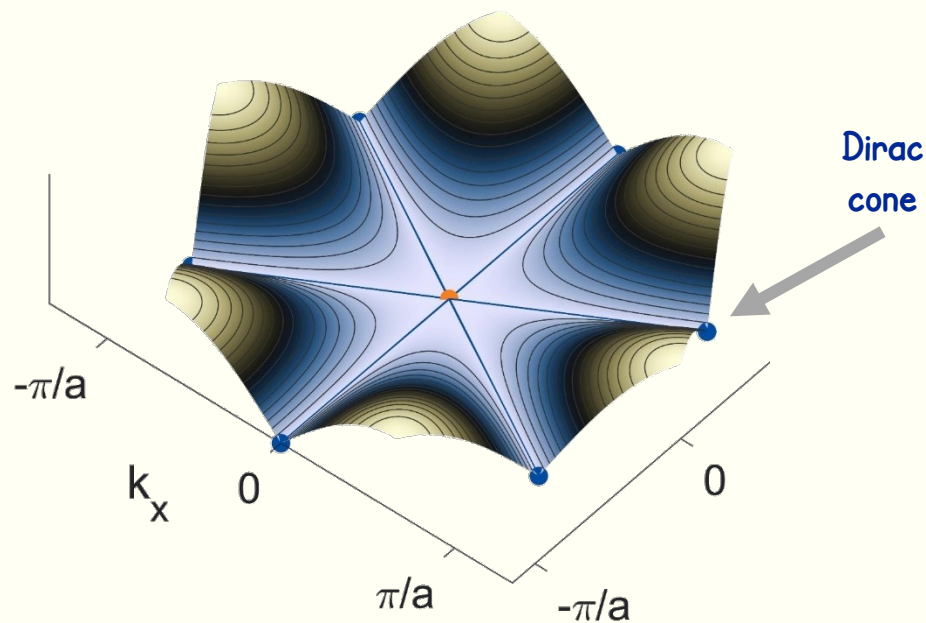
$$\gamma \dot{u} = h \nabla u \Rightarrow \omega \sim \left(\frac{h}{\gamma} \right) k = c k$$

waves and disturbances propagate without damping



SPECIAL CASE

Threefold symmetry of the hydrodynamic interaction matches the symmetry of a hexagonal crystal



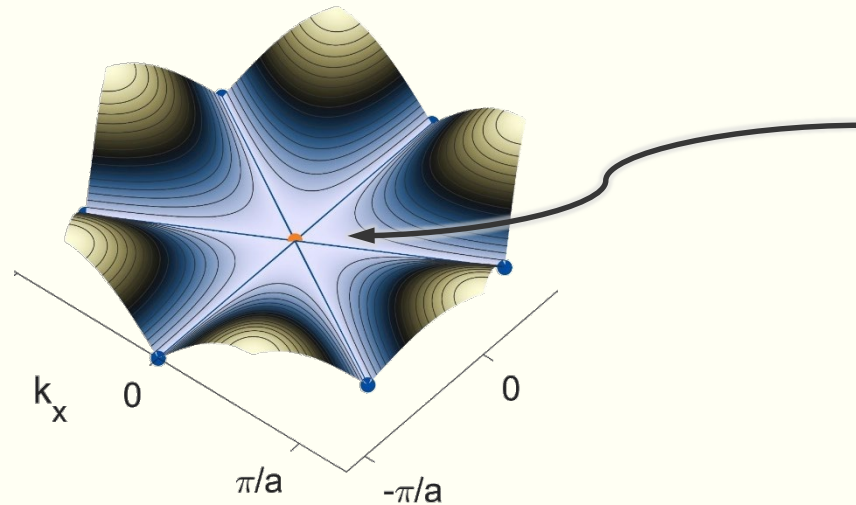
In a hexagonal lattice with nearest neighbors

$$\omega_k \sim \pm \left| \sin \frac{1}{2} k_x a \right| \left| \cos \frac{1}{2} k_x a - \cos \frac{\sqrt{3}}{2} k_y a \right|$$

HALLMARKS:

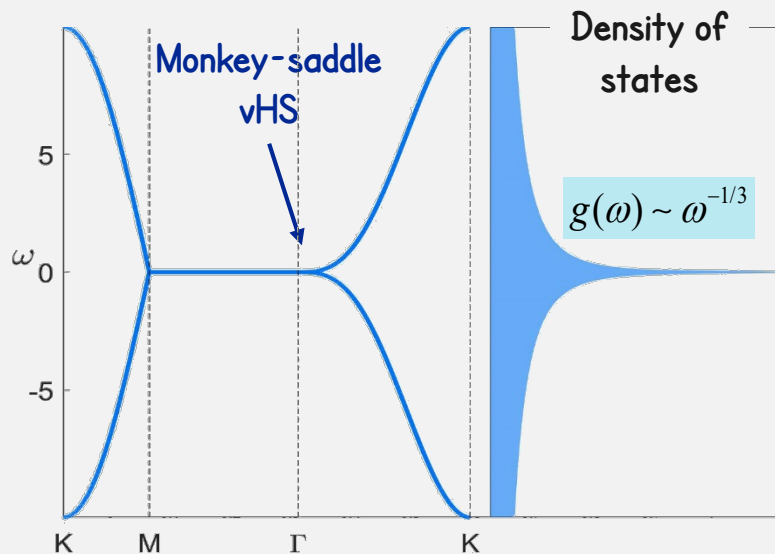
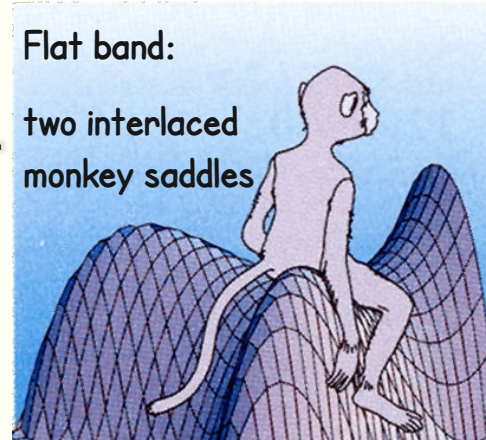
- Single van Hove singularity at $k=0$ (Γ)
- Dirac points (M) extend into 1D zero-frequency lines (cones $\rightarrow$ "wedges").

Hexagonal crystals exhibit a nearly-flat band of slow excitations



Flat band:

two interlaced
monkey saddles



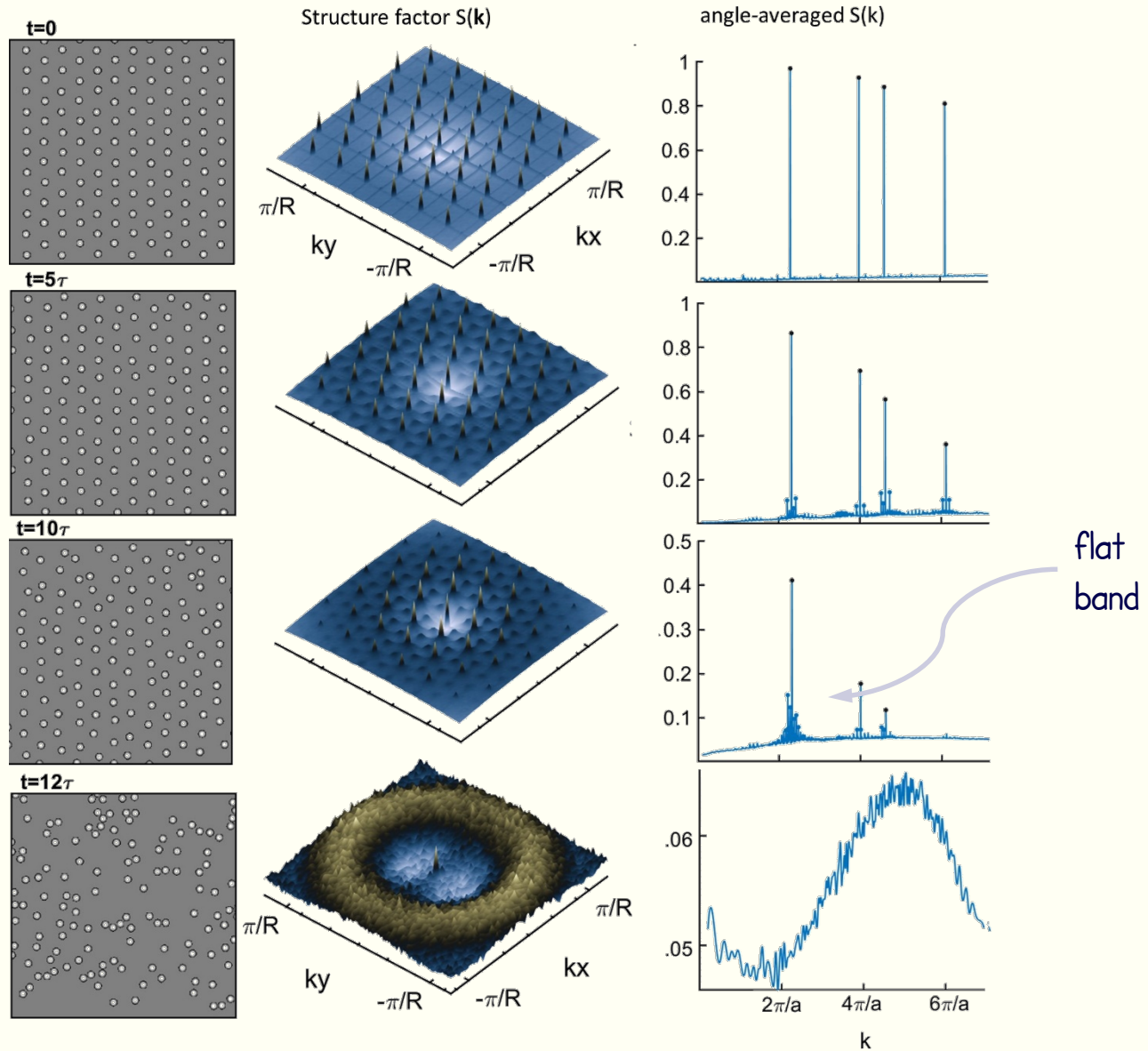
The van Hove singularity is a multicritical "monkey saddle" with divergent d.o.s.

$$\omega_{\mathbf{k}} \sim \pm k^3 \cos 3\phi$$

A flat band of long-wavelength slow modes with vanishing velocity and curvature

$$\frac{\omega_{\mathbf{k}}}{k} = \partial_{\mathbf{k}} \omega_{\mathbf{k}} = \partial_{\mathbf{k}}^2 \omega_{\mathbf{k}} = 0$$

Another path to disorder: melting of hexagonal crystals induced by flat band modes



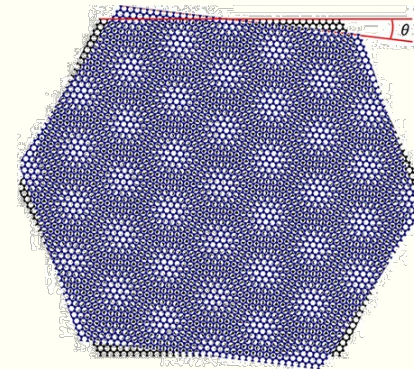
In flat bands slowing down of excitations leads to strong correlations

Structure factor $S(k)$



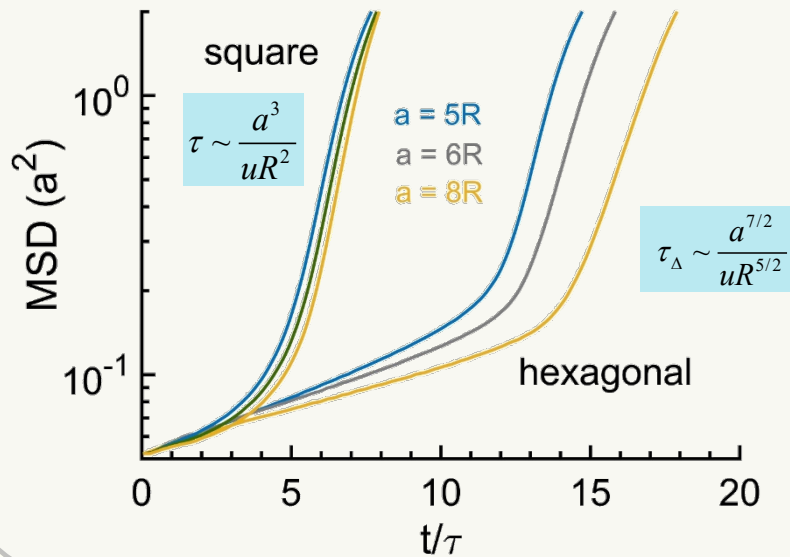
FLAT BANDS are associated with high- T_c superconductivity and topological insulators .

GRAPHENE bilayers exhibit flat bands and pairing when twisted at magic angle or buckled.

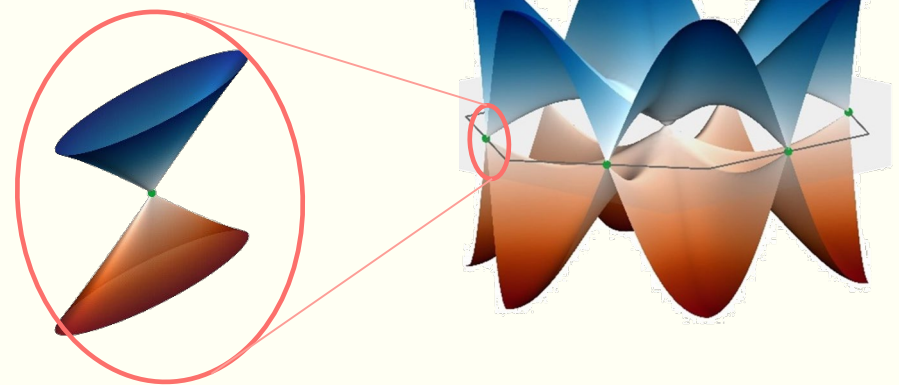
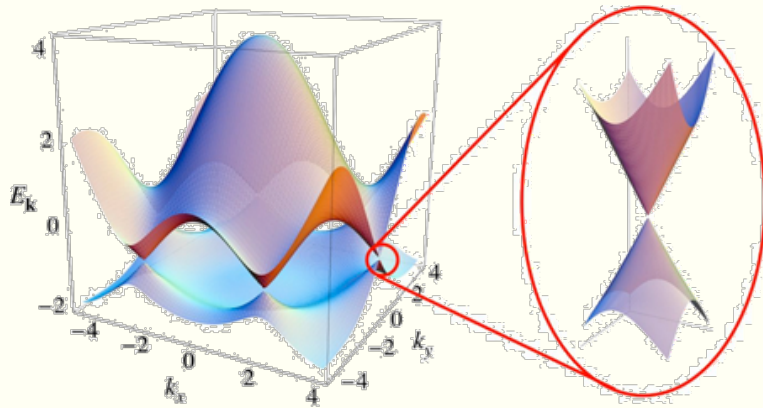


Flat-band-induced melting is a much sharper and collective non-eq. transition

Manifested also in **scaling** of disorder parameter



Certain similarities and dissimilarities to Graphene



[Castro Neto et al. 2011]

Graphene	Hydrodynamic crystal
Quasiparticles induced by symmetry of the honeycomb crystal	Quasiparticles from intrinsic parity symmetry of hydrodynamics
Pseudo-spin of electrons $\Psi_{\mathbf{k}}$	Polarization of phonons $\Psi_{\mathbf{k}}$
Dirac cones at the corners; van Hove at boundary.	Dirac points at the corners; van Hove inside Brillouin zone.
Flat bands by twisting bilayers at a magic angle	Flat bands by matching crystal and interaction symmetries

Non-Hermitian systems exhibit hallmarks unmatched by Hermitian counterparts

NON-HERMITICITY is *everywhere*:

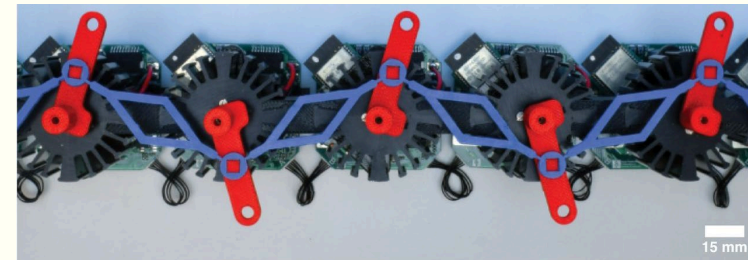
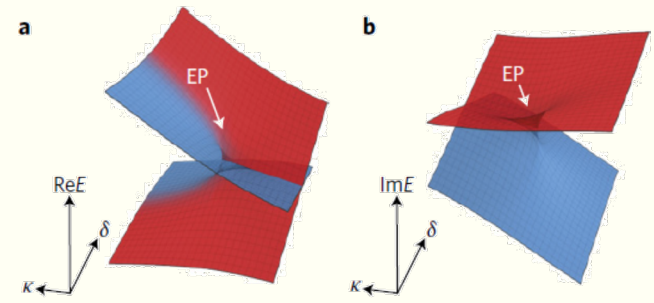
- most systems are **OPEN** and **Dissipative**.
- Any **EFFECTIVE**, coarse-grained system is open.

Exotic TOPOLOGICAL PHENOMENA:

- Skewed spectral bands, exceptional points, fractional charges, PT symmetry breaking

Photonic, phononic, electronic, atomic lattices...
recently in *active matter* (e.g. **ODD** viscosity and elasticity)

$$H \neq H^\dagger$$



See *Topological Active Matter*, Shankar et al. 2020 arXiv:2010.00364

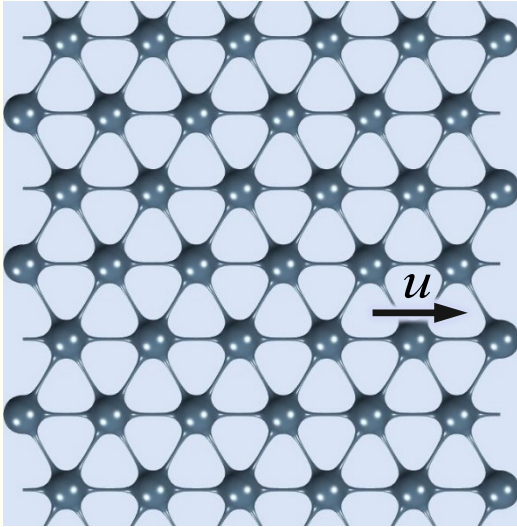


Can we observe exotic topological phenomena in more common settings?

...we are immersed in a dissipative, non-Hermitian world, and living systems are immanently open

SIMPLE ANSWER:

The interplay of elasticity and hydrodynamics induces non-Hermiticity



hydrodynamic dipoles

$$\mathbf{f}^{\text{hyd}}(\mathbf{r}) \sim \gamma u \frac{R^2}{r^2} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$$

$$\mathbf{f}(-\mathbf{r}) = \mathbf{f}(\mathbf{r})$$

+ springs (central conservative forces)

$$\mathbf{f}^{\text{elas}}(\mathbf{r}) \sim \kappa \Delta r \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

$$\mathbf{f}(-\mathbf{r}) = -\mathbf{f}(\mathbf{r})$$

The "hydro-elastic" number

$$\mathcal{E} = \frac{1/\tau_{\text{elas}}}{1/\tau_{\text{hyd}}} = \frac{\kappa a^3}{\gamma u R^2}$$

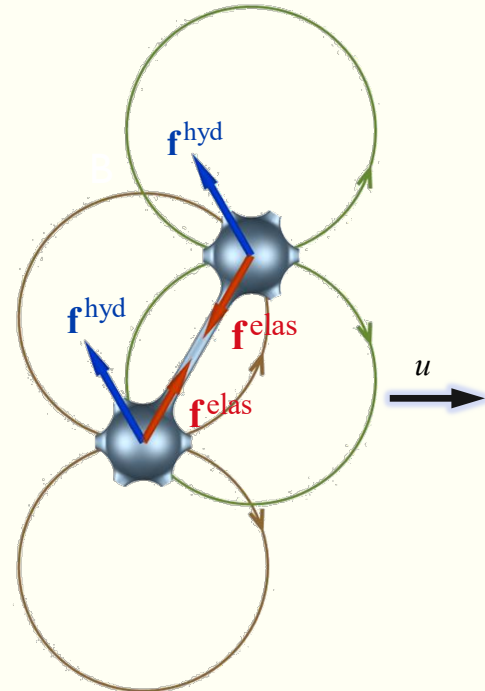
κ - spring constant

γ - drag coeff.

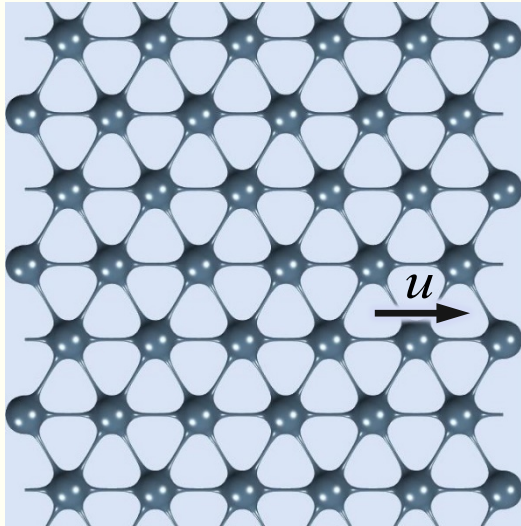
u - crystal velocity

a - lattice constant

R - particle size



Hydrodynamic and elasticity have opposite parity and Hermiticity



- Equations of motion
$$\gamma \dot{\mathbf{r}}_i = \mathbf{F} + \sum_j \left(\mathbf{f}_{ij}^{\text{hyd}} + \mathbf{f}_{ij}^{\text{elas}} \right).$$

- Normal modes:
$$\delta \mathbf{r}_i = \boldsymbol{\psi}_{\mathbf{k}} e^{i(\mathbf{k} \cdot \bar{\mathbf{r}}_i - \omega t)}.$$

- $\boldsymbol{\psi}_{\mathbf{k}}$ is eigenvector of
$$\omega_{\mathbf{k}} \boldsymbol{\psi}_{\mathbf{k}} = \mathbf{H} \boldsymbol{\psi}_{\mathbf{k}}$$
 with the hydro-elastic operator

$$\mathbf{H} = \mathbf{H}_{\text{hyd}} + \varepsilon \mathbf{H}_{\text{elas}}$$

$$\begin{pmatrix} \Omega_x \\ \Omega_y \end{pmatrix} \sim \sum \frac{e^{i\mathbf{k} \cdot \bar{\mathbf{r}}_j}}{\bar{\mathbf{r}}_j^3} \begin{pmatrix} \cos 3\theta_j \\ \sin 3\theta_j \end{pmatrix}$$

$$\mathbf{H}_{\text{hyd}} = \boldsymbol{\Omega} \cdot \boldsymbol{\sigma}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_l \end{pmatrix} \sim \sum e^{i\mathbf{k} \cdot \bar{\mathbf{r}}_j} \begin{pmatrix} \cos 2\theta_j \\ \sin 2\theta_j \\ 1 \end{pmatrix}$$

$$\mathbf{H}_{\text{elas}} = -i(\boldsymbol{\omega} \cdot \boldsymbol{\sigma} + \omega_l \cdot \mathbf{1})$$

- Hydrodynamics is **Hermitian** and odd under parity;
Elasticity is **skew-Hermitian** and **even**.

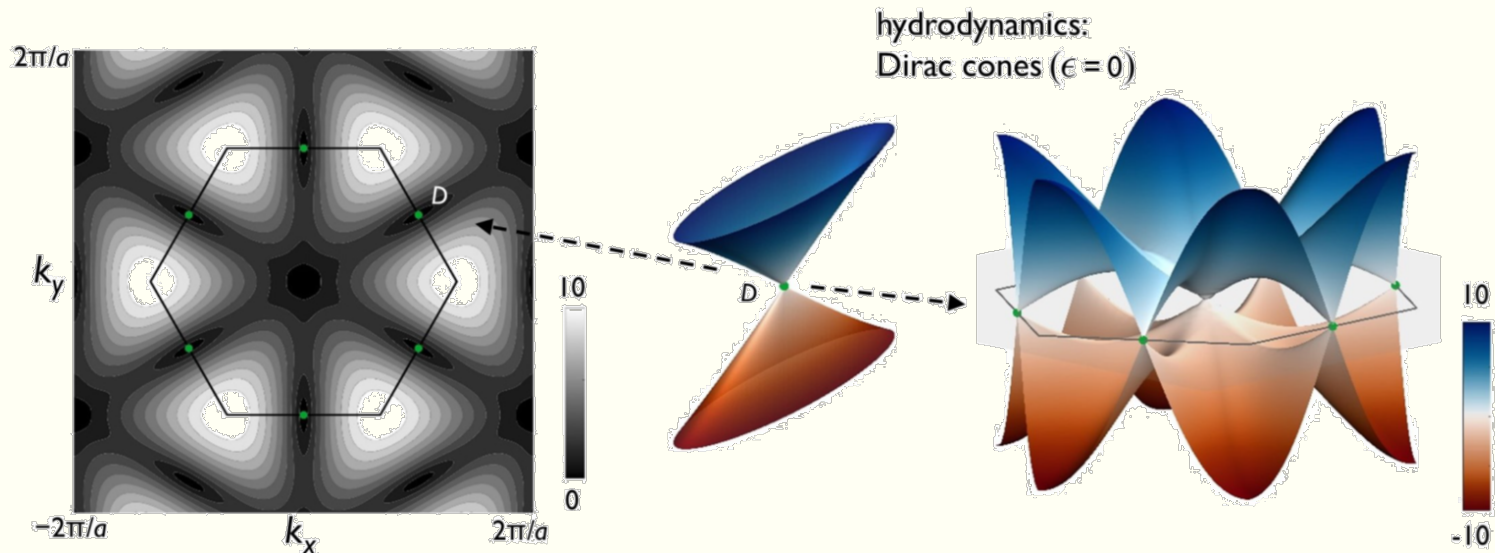
$$\mathbf{H}_{\text{hyd}} = \mathbf{H}_{\text{hyd}}^\dagger, \quad \mathbf{H}_{\text{hyd}}(-\mathbf{k}) = -\mathbf{H}_{\text{hyd}}(\mathbf{k})$$

$$\mathbf{H}_{\text{elas}} = -\mathbf{H}_{\text{elas}}^\dagger, \quad \mathbf{H}_{\text{elas}}(-\mathbf{k}) = \mathbf{H}_{\text{elas}}(\mathbf{k})$$



Questions ?

The purely hydrodynamic system shows real phonon spectrum with Dirac cones



The two bands of the spectrum

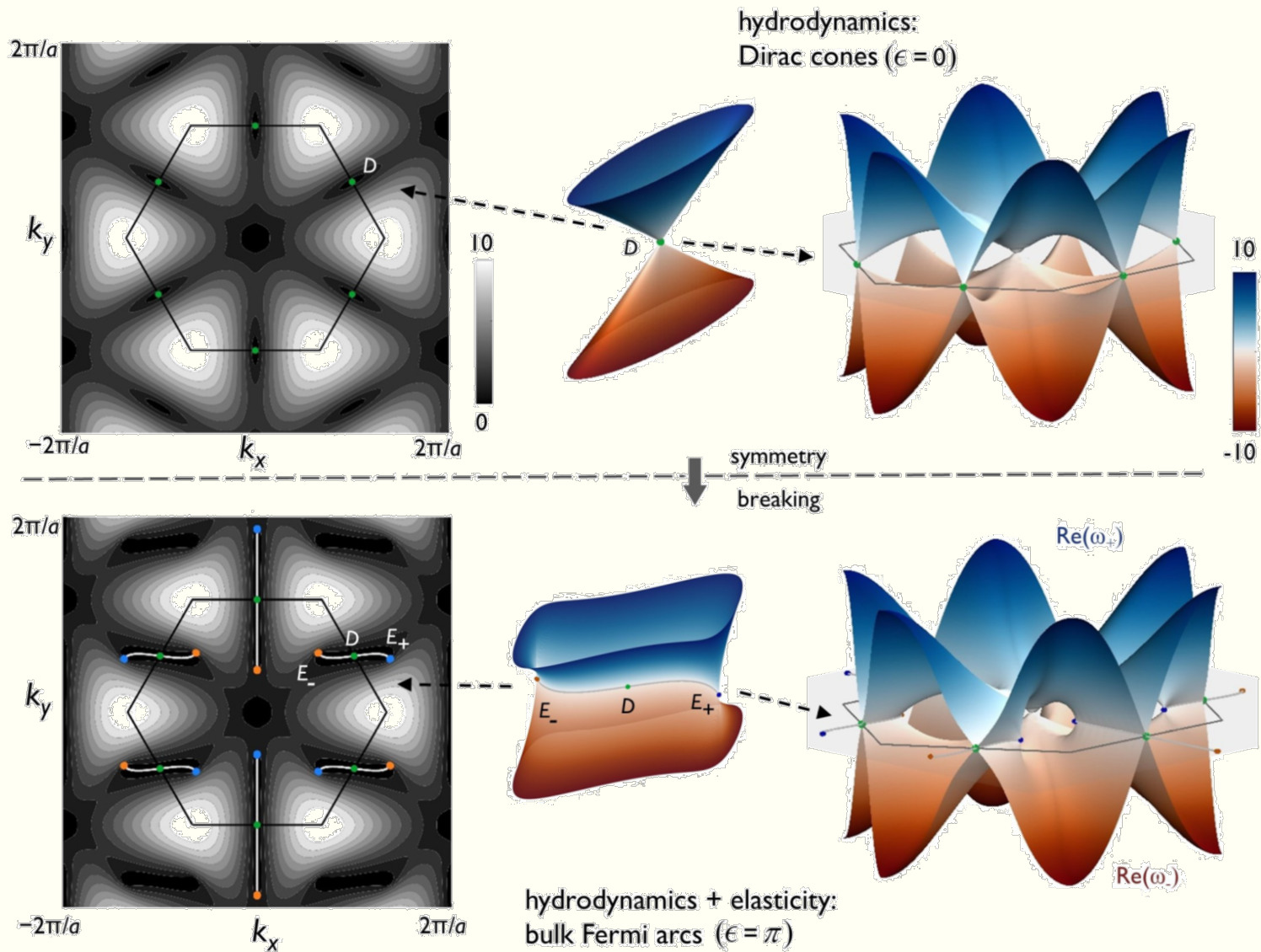
$$\omega_{\pm} = i\epsilon\omega_1 + \sqrt{\nu_+\nu_-}$$

$$\nu_{\pm} = (\Omega_x - i\epsilon\omega_x) \pm (\Omega_y - i\epsilon\omega_y)$$

Polarization eigenvectors

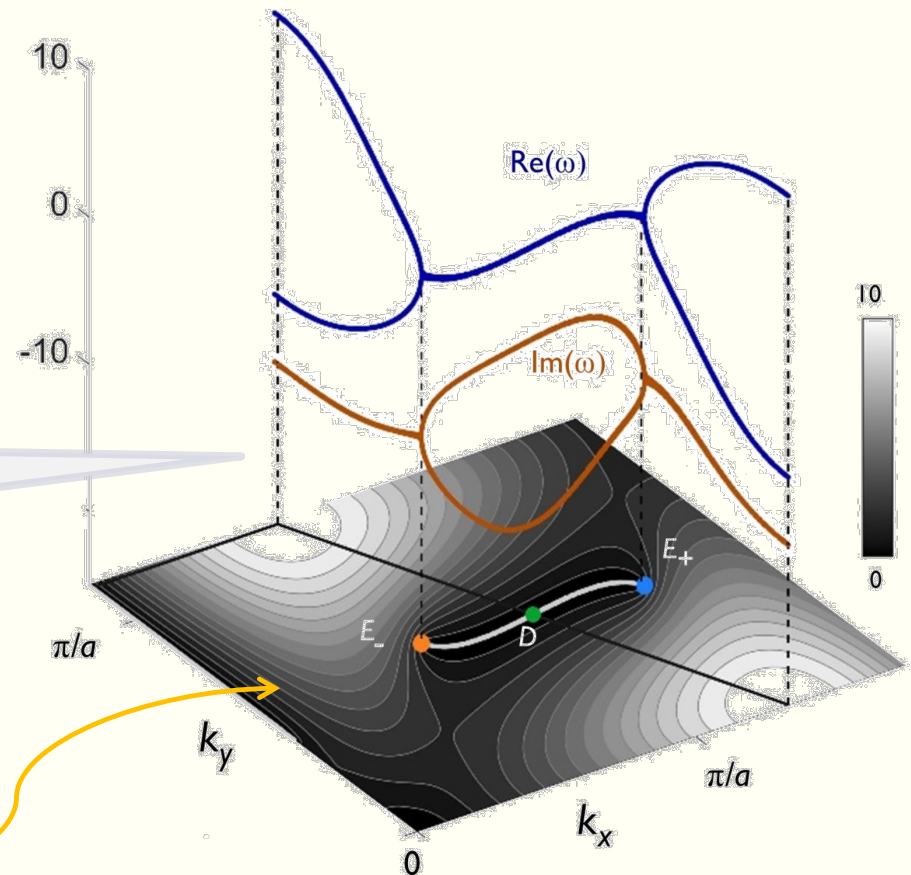
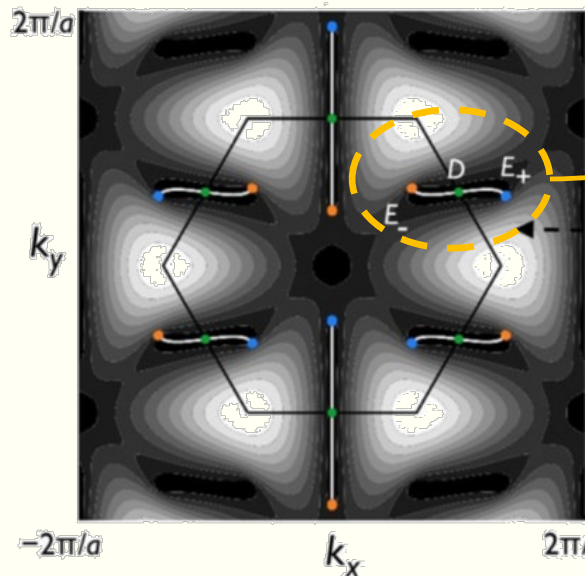
$$\Psi_{\mathbf{k}} \sim \begin{pmatrix} \sqrt{\nu_{\pm}} \\ \sqrt{\nu_{\mp}} \end{pmatrix}$$

Elasticity breaks the symmetry and splits the Dirac cones into Bulk Fermi Arcs



Bulk Fermi* Arcs are open-ended iso-frequency contours joining a pair of exceptional points

BULK FERMION* ARCS manifest non-Hermiticity in the *bulk* spectrum, unlike *surface* Fermi arcs induced by Weyl points in 3D Hermitian systems



At the **exceptional points**:

- Eigenvalues **and** eigenvectors coalesce.
- Bands are purely dissipative (real part = 0)
- **Square-root singularity** of the Riemann surface.



** No electrons involved... (as in photonic crystals)*

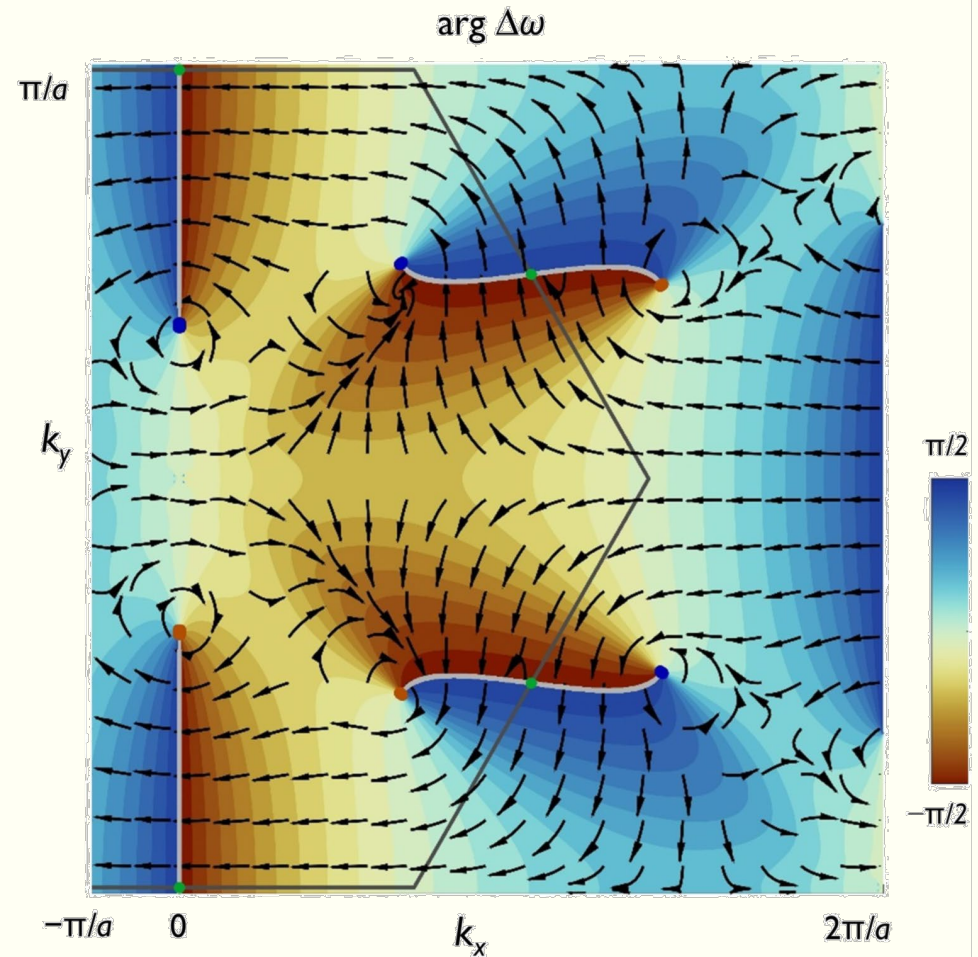
The exceptional points are topological defects with half-integer charges

Charges are found from the Berry phase

$$\nu = -\frac{i}{2\pi} \oint d\mathbf{k} \cdot \boldsymbol{\psi}_\mathbf{k}^* \nabla_\mathbf{k} \boldsymbol{\psi}_\mathbf{k} = \pm \frac{1}{2}$$

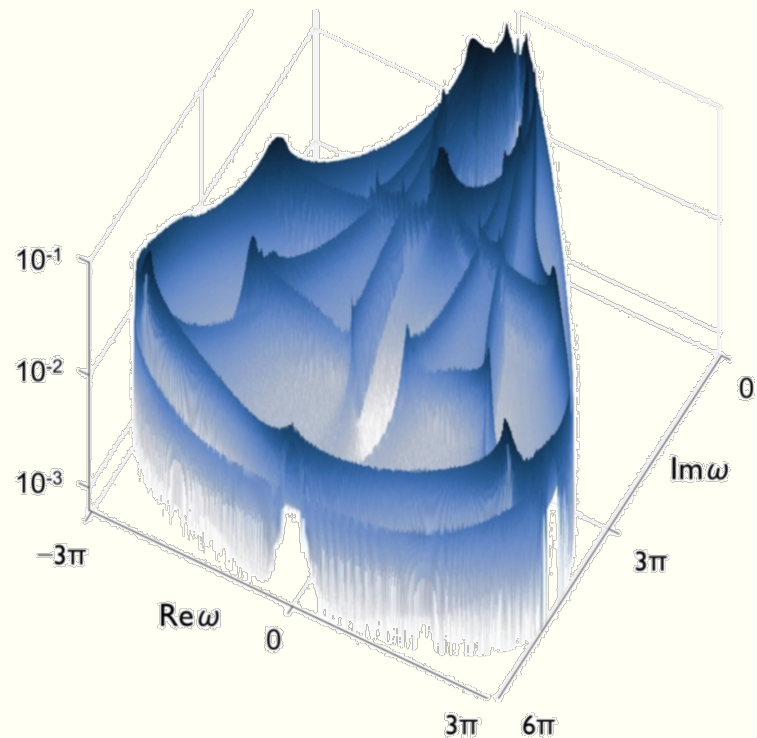
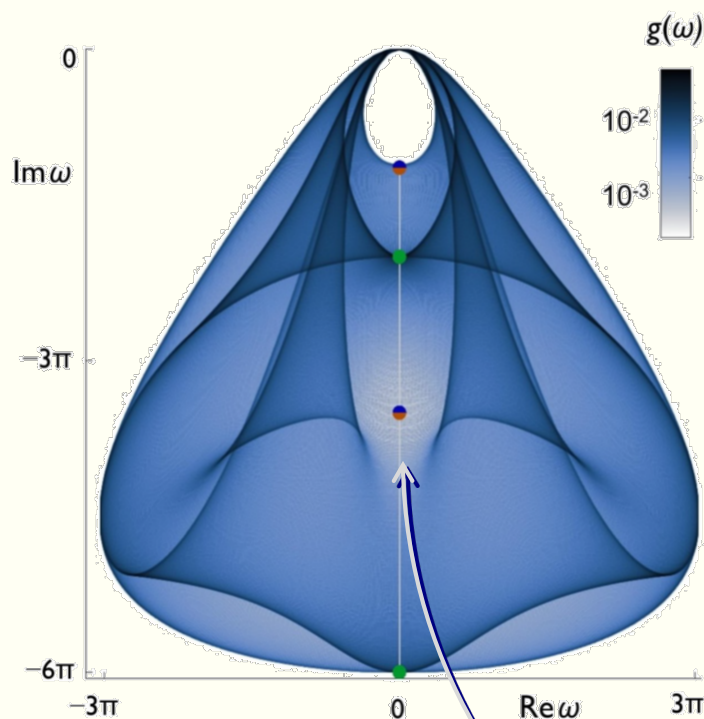
or other winding numbers, such as the
vorticity of the band gap:

$$\nu = -\frac{1}{2\pi} \oint d\mathbf{k} \cdot \nabla_\mathbf{k} \arg(\omega_+ - \omega_-) = \pm \frac{1}{2}$$



The density of states show extended van Hove "ridges" reflecting the topology

$$g(\omega) = \left(\frac{a}{2\pi} \right)^2 \int d^2\mathbf{k} \delta(\omega - \omega_{\mathbf{k}})$$

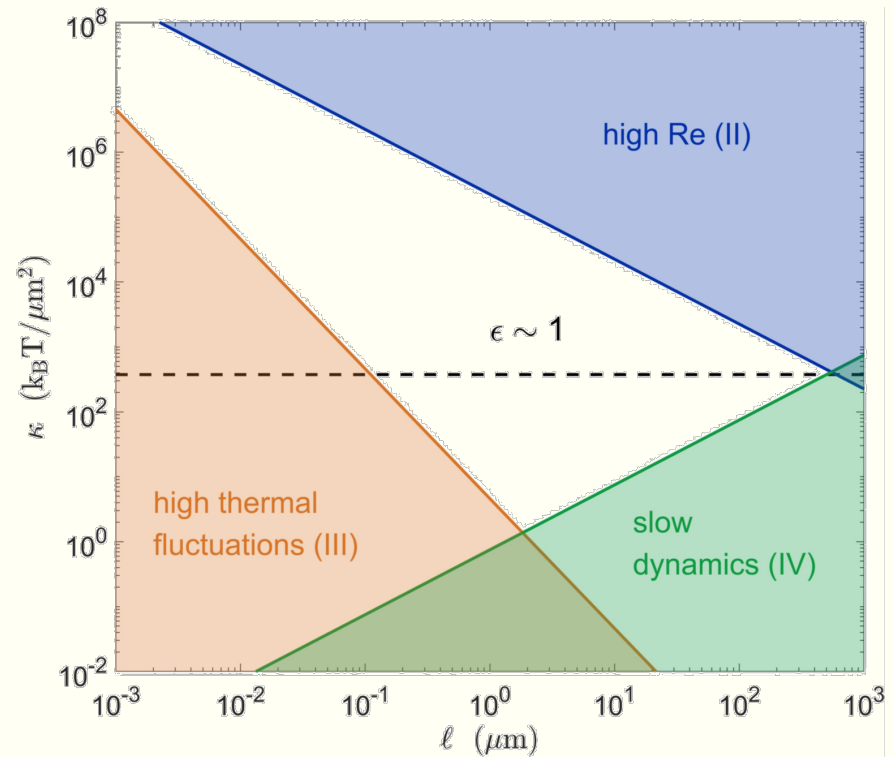


Level repulsion shows in low density of states around the exceptional points.

Proposal

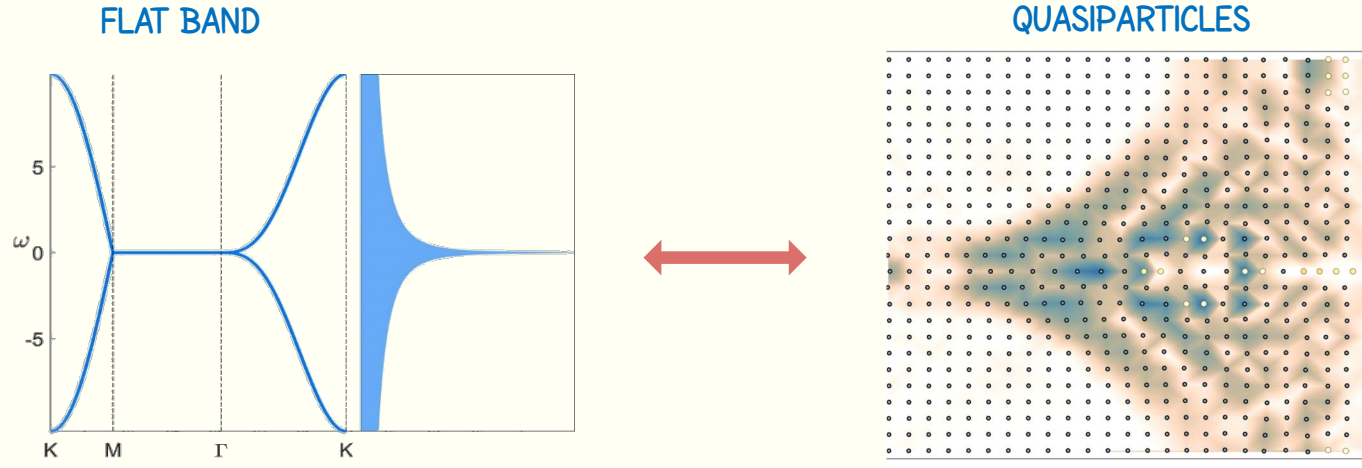
Exceptional topology can be observed in various soft matter systems subject to viscous flow

- Colloidal crystals connected by DNA springs or effective springs induced by magnetic repulsion...
- Also at the nanoscale: lattices made of DNA, protein or patchy nano-particles.
- Periodic lattices and low-Reynolds flows are *omnipresent* in biology and biotechnology.



Are non-Hermitian topological phenomena also widespread in these realms?

- Basic notions of quantum solid theory - QUASIPARTICLES and BAND THEORY - provide insight into the many-body dynamics of a classical dissipative system



- EXOTIC TOPOLOGY is common in ordinary soft matter (in flow)

THANKS

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